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AMT INTERMEDIATE

TEACHER'S GUIDE

Applied Mathematics for Today
Intermediate SI Metric Edition

Prepared by John Seymour

Dino Dottori · George Knill · John Seymour

The Ryerson Mathematics Program

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Applied Mathematics for Today
Intermediate SI Metric Edition

Dino Dottori
George Knill
John Seymour

The Ryerson Mathematics Program

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Table of Contents

Chapter 1	Graphing of Data From Investigations	2
Chapter 2	Algebra	8
Chapter 3	Equation of The Straight Line	21
Chapter 4	Distance Between Two Points	27
Chapter 5	Linear Systems	31
Chapter 6	Radicals and Exponents	38
Chapter 7	The Exponential Function and Logarithms	44
Chapter 8	Trigonometry	48
Chapter 9	Vectors in Two Dimensions	58
Chapter 10	Statics: Coplanar Forces	66
Chapter 11	The Circle	71
Chapter 12	Borrowing Money	76
Chapter 13	Compound Interest	81
Chapter 14	Buying and Selling, Profit and Loss	87
Chapter 15	Statistics	95
	Answers to Marginal Puzzles	104

AMT INTERMEDIATE TEACHER'S GUIDE

Introduction

The Review and Preview to each chapter provides exercises to review skills that will be required for the coming work plus some additional materials. Where skills are particularly pertinent it has been noted in this guide. It is not expected that every student will need to do all the Review/Preview exercises.

The General Comments to each chapter give a suggested time allowance. The first figure allows not more than 2 periods for the Review/Preview section, 1 period for testing and a little extra time for review. These figures total 172 periods. The second figure assumes that topics will have to be retaught from time to time and more time is allowed for review. These figures total 220 periods. If your class requires more than the lower suggested time figure some of the optional topics will have to be omitted.

The Section Comments are written in four parts.

A - The objectives.

B - A short opening exercise reviewing basic skills to be required by this section.

C - Comments and suggestions on teaching the topic.

D - Comments and suggestions on the exercise.

Where two or more sections deal with the same topic they have been covered together.

Chapter Tests, Notes and Answers are provided to give examples of possible test questions. The tests should be modified to reflect the areas of concentration selected by the teacher.

Review and Preview to Chapter 1

Exercise 1 has been suggested as an oral exercise to give an opportunity to quickly spot students who may be having arithmetic problems.

Exercises 2 and 4 are suited to calculator drills.

Exercise 3 will help students recall basic algebra skills.

Chapter 1 Graphing Data From Investigations

This chapter should be covered in 9 to 10 periods including Review Preview and Testing.

Chapter 1 leads through experimentation and the graphing of data to direct and inverse variation. It should involve considerable activity in the classroom and can be used to set an atmosphere of enquiry to be maintained throughout the year. This chapter can be sequenced after Chapter 2 if there is the desire to establish class routines before starting on group activities.

1.1 Graphing Data

A. Objectives - Review graphing
- Discover through experiment and graphical analysis examples of linear and non-linear relations.

B. Go directly to the investigations.

C. It is suggested that the class work in groups and each group be responsible for completing two or three investigations, at least one linear and one non-linear. Investigation 1.1 A linear relation. Students require

	a cloth measuring tape or string and metre stick, circular shapes and graph paper.
Investigation 1.2	Length to perimeter is linear. Length to area is non-linear (quadratic). Students require graph paper.
Investigation 1.3	Length to perimeter is linear. Length to area is non-linear (quadratic). Students require plain paper to draw triangles and graph paper. The triangle is used as the unit of area to simplify calculations. If this causes confusion an equilatural triangle with sides 1.52 cm has area approximately 1 cm^2.
Investigation 1.4	Length to area is non-linear (quadratic). Students require graph paper.
Investigation 1.5	Length to width is non-linear $\left(W=\frac{36}{\ell}\right)$ Length to perimeter is non-linear $\left(P= 2\,\frac{36}{\ell}+\ell\right)$. Students require graph paper.
Investigation 1.6	Stretch to mass is linear. Ideally $s \div m=k$ (k const.). Students require

	a metre stick, a spring and a set of uniform masses and graph paper.
Investigation 1.7	Initial height to rebound is linear. Ideally $h \div x =$ (K const.). A super ball or lacrosse ball works well. It takes a good eye to catch the rebound height. A metre stick and graph paper are also required.
Investigation 1.8	Period to pendulum length is non-linear. Ideally $t = k\sqrt{\ell}$ (K const.). Students require a mass (1 kg mass works well), string, watch and graph paper.
Investigation 1.9	If the size of the graduated cylinder is small relative to the size of the reservoir, volume to time will be almost linear. Otherwise the rate of flow decreases noticeably as the head decreases. This investigation can be messy. Students require container, graduated cylinder, watch, water and graph paper.
Investigation 1.10	Area to mass is linear. A permanent set of shapes may be cut from plywood or particle board. A simpler alternative is to have the students cut disposable shapes from heavy cardboard.

This requires a more accurate balance. Students require a set of shapes, balance and graph paper.

1.2 Relations and Functions

A. Objectives - The student can recognize and correctly use the terms relation, function, linear relation.

- Draw a graph from a given set of data and interpolate values from the graph.
- Begin to investigate rate of change as illustrated by the graph.

B. Data will have to be rounded off when setting up axes. Round off 4 763 961.652 to the nearest 1/10, 10, 100, 10 000.

C. Put a table of values on the board such as the following and check the mechanics of setting up a graph.

Day	1	2	3	4	5	6
Cost($)	4.80	7.50	9.40	12.30		8.60

Which values go along the horizontal axis?

How many squares are required for each axis?

Plot the points to represent the values and join them.

Interpolate for day 5.

D. When values for the independent variable are predicted beyond the range of known values it is called extrapolation. This occurs in question 1(a). Discourage attempts to estimate beyond reasonable expectations of accuracy. Questions 4,5 and 6 ask students to assess the rate of growth from the graph. A review of the concept of slope can flow from these.

1.3 Direct Variation

A. Objectives - The student recognizes that "y varies directly as x", $y \propto x$, $y=kx$ and $\frac{y}{x}=k$, (k const.) are equivalent statements.

- Solve problems involving direct variation.

B. Fill in the missing values

$\frac{2}{3} = \frac{4}{\quad} = \frac{6}{\quad} = \frac{\quad}{18} = \frac{\quad}{27} = \frac{\quad}{7.5} = \frac{2.4}{\quad}$ (6,9, 12, 18, 3, 3.6)

$\frac{3}{\quad} = \frac{\quad}{15} = \frac{27}{45} = \frac{54}{\quad} = \frac{\quad}{100} = \frac{1.5}{\quad} = \frac{\quad}{0.5}$ (5, 9, 90, 60, 2.5, 0.3)

C. Stress the idea that if the relation is direct variation the division $\frac{y}{x}$ for any pairs of corresponding values gives the same answer. Therefore we have a constant slope and a straight line graph.

Calculators can be used to stress the idea of a constant by calculating $k = \frac{y.}{x.}$, for x. and y. corresponding values of x and y and placing the value for k in memory. Then $y = kx$ or $x = y \div k$ for other pairs.

D. The problems in attempting to be real life are idealized. A discussion could be held concerning factors that lead to inaccuracies in the predicted answers.

1.4 Partial Variation

A. Objectives - The student can set up a linear relation of the form $C = aN+b$ and solve for C when a,b and N are known.

- Apply these skills to problem situations.
- Graph two or more relations and select areas

of optimum value.

B. Complete the tables

X	0	1	2	3	4
Y					

if $y = 3x+5$
(5, 8, 11, 14, 17)

X	0	2	4	6
Y				

if $y = \frac{1}{2}x-1$
(-1, 0, 1, 2)

C. Get examples from the class that illustrate a situation where the cost is the sum of a constant plus a variable.

Automobile - Fixed annual costs - license, insurance, payments. Costs related to kilometres driven - gas, oil, maintenance.

Follow this with a situation where the student can select the most economical approach to a purchase.

Example - A golf course has season memberships for $300. or pay-as-you-play for $12 for 18 holes. How many times must a golfer play 18 holes before it is more economical to purchase a membership? (26). What other considerations might be taken into account? (lockers, dining privileges etc.)

D. In exercise 1-4 each problem involves a constant quality plus a variable quantity. Questions 1 to 4 can be done by numerical calculation. Questions 5 and 6 require algebra, trial and error or graphs. Graphs are recommended.

Test Chapter 1

A sheet of graph paper is required.

1.

Speed (Km/h)	180	90	60	45	36
Time (h)	2	4	6	8	10

(a) Plot the points and join them with a smooth curve.

(b) Find the speed when the time is 5h.

(c) Find the time when the speed is 50 Km/h.

2. If Y varies directly as X complete the table.

X	3	4	6	8	9	12
Y		6				

3. The cost of repairing an auto is $18/h labour plus the cost of parts. If the time for the repair was 2.5h and the cost of parts was $76 what was the total cost.

4. The cost of a purchase of wallpaper varies directly as the number of rolls. If 5 rolls cost $37.50 find the cost of 10 rolls, of 7 rolls.

Test Chapter 1 Comments and Answers

Part of the evaluation for this chapter could be on reports from the investigations. The test covers sections 1.2 to 1.4.

Answers

1. (b) 72 km/h (c) 7.2h

2.

X	3	4	6	8	9	12
Y	4.5	6	9	12	13.5	18

3. Cost $121.00

4. 10 rolls cost $75.00, 7 rolls cost $52.50.

Review and Preview to Chapter 2

Exercises 1 and 4 review work with decimals and may be treated as calculator exercises.

Exercises 2 and 3 review operations on monomials which will be required in the chapter.

Chapter 2 Algebra

This chapter should be covered in 20 to 23 periods including Review and Preview and Testing.

Students at this stage and level seem to vary greatly in their ability to do algebra. If progress is slow treat sections 2.3 and 2.8 as optional.

Tests are provided after sections 2.5 and 2.10.

2.1 Product of a Monomial and a Polynomial

A. Objectives
- The student can multiply a binomial or a trinomial by a monomial.
- Where the question involves a sum or difference of products, simplify the resulting expression.
- Recognize this as an application of the distributive property of multiplication over addition.
- *Apply this skill to the determination of areas of rectangles, triangles, etc. where the lengths of the sides are represented by algebraic expressions.

* optional

B. Simplify and Compare the following products:

1.	$-(3)(4)$,	$(-3)(4)$	(-12)
2.	$-(5)(-6)$,	$(-5)(-6)$	(30)

3.	$-(-8)(-3)$,	$(8)(-3)$	(-24)
4.	$-(-7)(2)$,	$(7)(2)$	(14)
5.	$-\frac{1}{2}(-10)$,	$(-\frac{1}{2})(-10)$	(5)

C. Questions such as $x(3-2x) \; -5(2+x)$ are commonly handled in one of two ways.

Leads more directly to $= (3x-2x^2) - (10+5x)$

products of polynomials $= \boxed{3x-2x^2-10-5x}$

for students having difficulties $= -2x^2-2x-10$

or mental step

A better approach for $= \boxed{x(3-2x)+(-5)(2+x)}$

more able students $= 3x-2x^2-10-5x$

$= -2x^2-2x-10$

D. The questions dealing with areas are optional but they afford an opportunity to review area formula, to emphasize that algebraic expressions represent numbers, and they practice skills that will be used in more advanced problem solving. This is a good topic for student generated problems

2.2 Product of Two Binomials

A. Objective - The student can multiply two binomial expressions of the first degree.

- Recognize the square of a binomial as a special case.
- Recognize the product as an application of the distributive property.
- *Apply this skill to the determination

of areas of rectangles, triangles, etc. where the lengths of the sides are represented by algebraic expressions.

- *Apply the distributive property to products involving trinomials.

*optional

B. Simplify

1. $3x(2x+4)+5(2x+4)$ $(6x^2+22x+20)$
2. $x(3x+1)-2(3x+1)$ $(3x^2-5x-2)$
3. $4a(3a-2b)=5b(3a-2b)$ $(12a^2-23ab+10b^2)$

C. Although $3(2x+4)(5x-6)$ may be expanded by either $[3(2x+4)](5x-6)$ or $3[(2x+4)(5x-6)]$ the second sequence has the advantage of having the students working with smaller numbers for the more difficult operations.

D. This exercise requires the orderly application of the skills learned so far in the chapter. Students who are experiencing frustration at this point can sometimes be helped by having them simplify the questions in parts.

Example Exercise 2-3 question 2(a)

$$\begin{array}{lll}
2(2x-1) & 3(x+2)(x-4) & 2(2x-1)+3(x+2)(x-4) \\
= 4x-2 & = 3(x^2-4x+2x-8) & = 4x-2+3x^2-6x-24 \\
 & = 3(x^2-2x-8) & = 3x^2-2x-26 \\
 & = 3x^2-6x-24 & = 3x^2-2x-26
\end{array}$$

2.4 Division of a Polynomial by a Monomial

A. Objectives - To review the division of monomials.

- The student can divide a polynomial by a monomial.

- Recognize that variables may not take values such that the denominator equals zero.

B. Simplify

1. $12 \div 4$ 2. $\frac{12}{4}$ 3. $\frac{-15}{3}$

4. $\frac{24}{-6}$ 5. $-42 \div (-6)$

6. $\frac{(12)(-35)}{(-21)(10)}$ 7. $\frac{(-16)(55)}{(-22)(-20)}$

Answers. 3, 3, -5, -4, 7, 2, -2

C. Have the students consider the separation line of the fraction a bar bracket.

If $3x(9x^2+6x)$ means multiply every term in the bracket by 3x

then $\frac{9x^2+6x}{3x}$ means divide every term over the bar bracket by 3x. Since multiplication is closed on the set of integers the first case can always be expanded, division is not, so the second expression may have to be left. An alternative with expressions such as $\frac{9x^2+6x+4}{3x}$ is partial simplification to $3x+2+\frac{4}{3x}$.

D. In the C exercise the questions have been chosen so that it is possible to simplify each fraction before adding or subtracting so that the problem of common denominators does not arise.

2.5 Division of a Polynomial By a Binomial

A. Objectives - The student can divide a polynomial by a monomial.

- Recognize that division is not always exact and can express an answer that contains a remainder.
- Can determine values for the variable that cause the expression to be undefined by reason of a zero denominator.
- Can handle expressions where a power in the dividend has zero coefficient or is missing.

B. 1. Divide 559 by 36. $\left(15 \ \frac{19}{36}\right)$

2. Arrange the following expressions in descending order of powers.

(a) $4x^3-3x+9x^4-7+2x^2$ (b) $6a-5a^6+7a^3-4a^5+9$

C. Students who have a near phobia about long division with whole numbers will probably have to be walked slowly through this topic. Two possible approaches that help:

1. Give the students a sequence of partially-finished questions to complete.

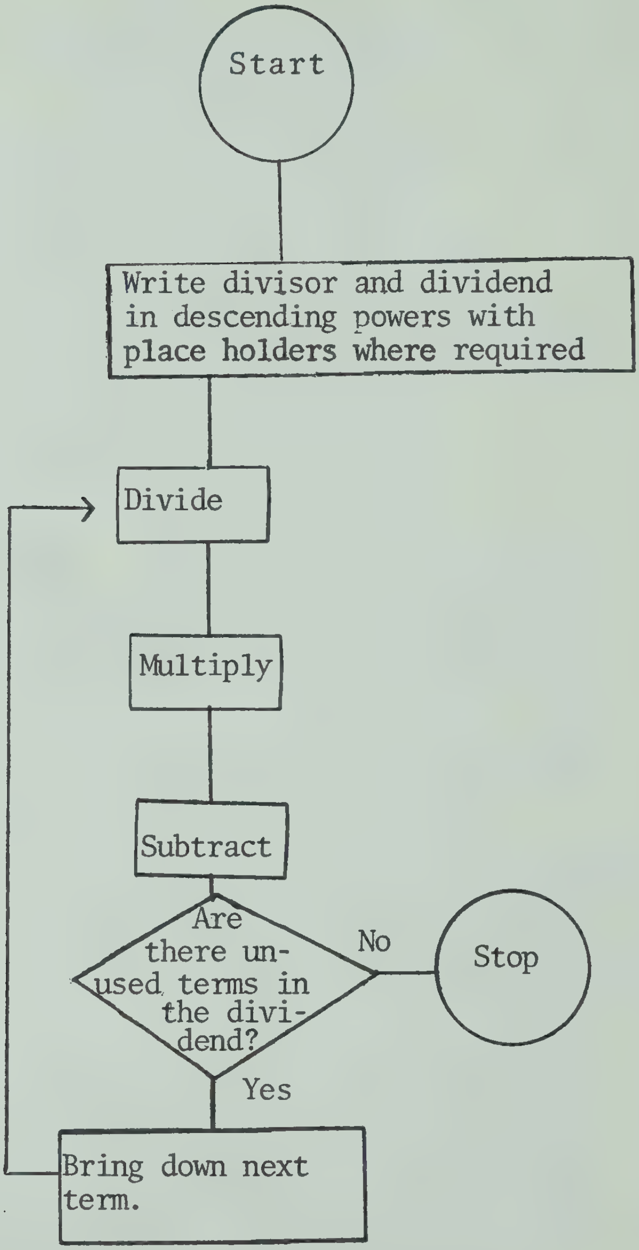
(a)
$$\begin{array}{r l} & x+10 \\ x-4 \,\big) & \overline{x^2+6x-10} \\ & \underline{x^2-4x} \\ & \quad 10x-10 \\ & \quad \underline{\text{▨} \quad \text{▨}} \\ & \qquad \underline{\text{▨}} \end{array}$$

(b)
$$\begin{array}{r l} & x \ \text{▨} \\ x+3 \,\big) & \overline{x^2+5x+6} \\ & \underline{x^2+3x} \\ & \quad 2x+\text{▨} \\ & \quad \underline{\text{▨} \quad \text{▨}} \\ & \qquad \text{▨} \end{array}$$

(c)
$$\begin{array}{r l} & \text{▨} + \text{▨} \\ 2x-3 \,\big) & \overline{4x^2-2x+12} \\ & \underline{4x^2-6x} \\ & \quad \text{▨} \quad \text{▨} \\ & \quad \underline{\text{▨} \quad \text{▨}} \\ & \qquad \text{▨} \end{array}$$

(d) Now try one from the beginning.

2. Construct a flowchart


Start
Write divisor and dividend in descending powers with place holders where required
Divide
Multiply
Subtract
Are there unused terms in the dividend?
No
Stop
Yes
Bring down next term.

D. Most problems in the exercise will arise from forgetting to put the expressions in the proper order or not inserting place holders.

2.6 - 2.7 Linear Equations Part I and Part II

A. Objectives

- The student recognizes that solving an equation involves finding values for the variable that make the left side equal the right side and can demonstrate this by selecting the roots for a given equation from a set of numbers.
- Can solve linear equations with integral coefficients where the variable appears on both sides and where removal of brackets is required.
- Can solve equations with fractional coefficients by selecting a common denominator and multiplying through.
- *Can solve simple linear equations with literal coefficients
- Can check solutions by substitution into L.S. and R.S.

*optional

B. For each of the following equations select the elements from the given set that belong in the solution set.

1. $X+4=6$, $\{0,1,2,3,4,\}$ $\{2\}$
2. $2x+4=x-3$, $\{-10,-7,-5,0,3\}$ $\{-7\}$
3. $4x+6=2(2x+3)$, $\{-2,-1,0,1,2,\}$ $\{\text{all values}\}$
4. $x^2+4x-12+0$, $\{-6,-3,0,2,4\}$ $\{-6,2\}$

C. In solving equations the method is to get the equation into the form $x=a$ where a is the root or answer. The following sequence is helpful in most cases.

1. Remove fractions by multiplying through by the common denominator. Where decimals are involved multiplication by 10, 100 etc. though not necessary seems to make the solution appear simpler.
2. Remove brackets by multiplication. Remember expressions such as $x+2-(3x+4)$ may be treated as $x+2+(-1)(3x+4)$.
3. Simplify both sides by addition or subtraction of like terms.
4. Reduce missplaced terms to zero by adding the opposite to both sides, variables removed from R.S. absolute terms from L.S. Simplify as you go.
5. Reduce the coefficient of the variable to one by dividing both sides by the coefficient.
6. Check by substituting the answer into both sides.

D. Exercise 2-6 Part A and Exercise 2-7 Part A might be attempted orally before covering the main examples. If the exercise causes trouble remedial action will be required before progressing.

2.8 Equations Involving Polynomials

A. Objectives - The student can apply the skills of simplifying polynomials to the solution of equations. (Note that the examples and exercises have been constructed to simplify to linear equations.)

B. Simplify.

1. $(x+3)(x-4)$ 2. $(x-4)(x+1)$

3. $(2x-6)^2$ 4. $2x(3x-4)^2$

Answer. x^2-x-12, x^2-3x-4, $4x^2-24x+36$, $18x^3-48x^2+32x$

C. If you have had the students do the above four questions or similar ones as a refresher for their multiplication skills, ask, "For what values of x would the expression in question 1 equal the expression in questions 2?"

D. Students problems will arise when mechanical errors lead to quadratic equations and the question will arise as to whether this type of equation can be solved. Depending on the class, this might be a good time to allow yourself to be drawn "off topic" to the solution of simple quadratic $x^2=25$, $(x-2)^2=36$ $(x+4)(x-3)=0$.

2.9 Solving Inequations

A. Objectives - The student can solve inequations to the degree of complexity that they can handle in equations.

- Solutions can be expressed as a simplified statement such as $x \geqq 4$, or as a graph.
- Special attention is given to the reversal of sense when multiplication or division by negative quantities is applied to both sides.

B. Graph the following sets.

(a) $\{x/4>4, x\in R\}$

(b) $\{x/4<-1, x\in R\}$

(c) $\{x/4\geqq-2, x\in R\}$

(d) $\{x/4\leqq3, x\in R\}$

C. The coverage of this topic should be tailored to reinforce the students ability to solve equations and should appear to be a natural off-shoot from it rather than a completely new topic.

Example - Recall question 4(a) P.33

$4x-7=2x+3$ solution $x=5$

What happens if $4x-7>2x+3$? answer $x>5$

Check solution by substitution.

Let's look at question 4(d)

$4a-3=6a-11$, solution $a=4$

What happens if $4a-3>6a-11$ answer $a>4$
$a<4$ (wrong)

Check solutions by substitution.

Why was this one reversed? etc.

D. Select questions consistent with the students proven ability to solve the corresponding equations.

2.10 Formula Solving and Substitution

A. Objectives - The student can substitute values into a formula and apply their skills of equation solving to determine the corresponding value for the remaining variable.

B. Find the value for y in each case.

$y=ax+b$, $a=4$, $b=-2$, $x=3$ (10)
$y=x^2-3c$, $c=2$, $x=-5$ (19)
$y=\pi r^2$, $\pi=3.1$, $r=2.4$ (17.856)

C. If students have worked with equations with literal coefficients they should be encouraged to try method II. Most will be more comfortable with method I.

D. Encourage the students to identify the formula in the exercise.

Test Chapter 2 Part I Simplification Involving Products

1. Simplify

(a) $4(2x-1)$ (b) $6x-3(2x-5)$

(c) $3(5x-3y)+2(2x+4y)$ (d) $7x(3x^2+2x-4)$

(e) $2x(3x-4y)-5y(x+3y)$

2. Simplify

(a) $(3x+2)(2x-5)$ (b) $(2a+1)^2$

(c) $(4a+3b)(a-4b)$ (d) $3(x-6)(2x+1)$

(e) $(x-2y)(3x+y)-(2x+3y)(4x-5y)$

(f) $3(m+2n)(2m-5n)+(4m+7m)(4m-7n)$

3. Simplify

(a) $\frac{15a}{3}$ (b) $\frac{30x^2y}{6xy}$ (c) $\frac{-12abc^2}{-4ac}$

(d) $\frac{3x-6y}{3}$ (e) $\frac{25mn-30m^2}{-5m}$ (f) $\frac{-3x^3y^5+12x^2y^7}{-3x^2y^4}$

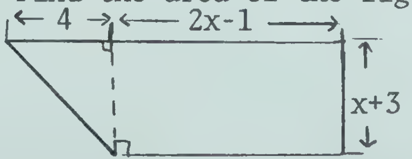
4. Divide

(a) $(2x^2+5x-12)\div(x+4)$

(b) $(3x^3+23x-14x^2-10)\div(3x-2)$

5. Optional Bonus

(a) Find the area of the figure



(b) Simplify $(x+3)(3x^2-4x+2)-(2x-1)(x^2+3x-5)$

(c) Divide $(3x^3+23x^2-10+24x)\div(x^2+6x-2)$

Test Chapter 2 Part II Equations and Inequations

1. Solve

(a) $x-3=9$ (b) $3x=-27$

(c) $2x+4=5$ (d) $6x+5-(2x+3)=22$

2. Solve and check by substitution

   (a) $4x+3=3x-7$ (b) $4(y-3)+2=7y-4$

3. Solve

   (a) $\frac{x}{4}=-5$ (b) $\frac{2x}{3}+\frac{1}{2}=\frac{9}{2}$

   (c) $\frac{3x-1}{7}=5$ (d) $\frac{3}{4}(x+2)+\frac{1}{2}(3x+1)=-10$

4. Solve and graph the solution set

   (a) $\{x/3x+1<10, x\in R\}$ (b) $\{x/3(x-2)\leqq 5x+4, x\in R\}$

5. (a) If $S=\frac{n}{2}(a+1)$ find n when $S = 2310$
   $a=4$ and $1=150$

   (b) If the solution for the equation $3x+4=5x+m$ is $x=-6$.

   What number must m represent.

6. Optional Bonus

   (a) Solve and graph (a) $\frac{y+7}{4}\leqq\frac{2y-1}{3}$ (b) Solve for x
   $y=mx+b$

Tests Chapter 2 Comments and Answers

Part I covers sections 2.1 to 2.5

Part II covers sections 2.6 to 2.10

Questions covering C exercise material or the most difficult topics have been noted Optional Bonus.

Answers Part I

1. (a) $8x-4$, (b) 15 (c) $19x-y$ (d) $21x^3+14x^2-28x$

   (e) $6x^2-13xy-15y^2$

2. (a) $6x^2-11x-10$, (b) $4a^2+4a+1$, (c) $4a^2-13ab-12b^2$

   (d) $6x^2-33x-18$, (e) $-5x^2-7xy+13y^2$, (f) $22m^2-3mm-79m^2$

3. (a) $5a$, (b) $5x$, (c) $3bc$, (d) $x-2y$, (e) $-5n+6m$

   (f) $xy-4y^3$

4. (a) $2x+3$ (b) x^2-4x+5

5. (a) $2x^2+7x+3$, (b) $x^3-2x^2-3x+11$, (c) $3x+5$

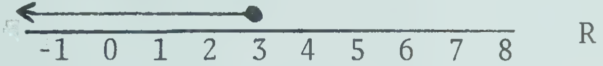
Answers Part II

1. (a) $x=12$, (b) $x=-9$, (c) $x=\frac{1}{2}$, (d) $x=5$

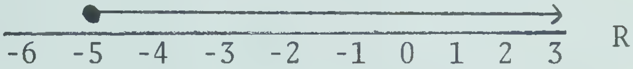
2. (a) $x=-10$, (b) $y=-2$

3. (a) $x=-20$, (b) $x=6$, (c) $x=12$, (d) $x=-\frac{16}{3}$

4. (a) $x<3$

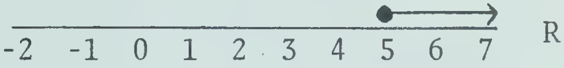


(b) $x \geqq -5$



5. (a) $n=30$ (b) $m=16$

6. (a) $y \geqq 5$



(b) $x=\frac{y-b}{m}$

Review and Preview to Chapter 3

This exercise reviews operations with mixed numbers. Exercise 1 has application to the work with slopes to be covered in this chapter. The rest of the Review and Preview stands alone and can be taken or omitted according to the needs of the students.

Chapter 3 Equation of the Straight Line

This chapter should be covered in from 10 to 12 periods including Review Preview and Testing.

It is recommended that the students be encouraged to analyze problems and check their work through the use of rapidly sketched graphs.

3.1 Slope

A. Objectives - The student can define slope as rise/run

- Becomes familiar with $\frac{\Delta y}{\Delta x}$ notation

- Can determine the slope of the line segment joining two points given their coordinates
- Recognizes that the slopes of all segments in a straight line are equal.

B. Plot the following pairs of points and tell how far apart they are.

1. (3,7), (3,10) 2. (-4,6),(-1,6) 3 units, 3 units
3. (5,-2),(5,5) 4. (9,3),(0,3) 7 units, 9 units

What do the pairs of points all have in common?

How could you determine the distances without graphing.

C. It is suggested that Investigation 5.1 follow the B exercise.

After taking up the ideas in examples 1 to 3 Pages 46,47 the students will require practice.

The investigation brings out important ideas about slopes that should be discussed with further examples.

D. Question 2 requires follow-up time to expand on the implications in determining slope.

Students should be encouraged to try questions 5,6,7 in the C exercise using a ruler and a piece of graph paper to get a graphical solution.

3.2 The Equation of a Line

A. Objectives - The student recognizes that a line is a set of points such that the slopes of all possible pairs of points from the set are equal.

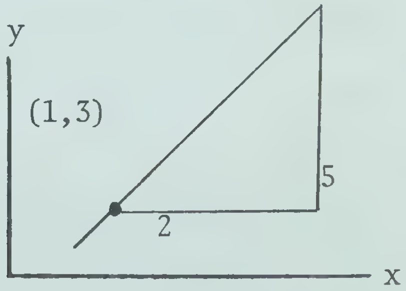
- A line is named by a linear equation that describes the relation between the coordinates of each point on the line.

- Can use the point-slope form $y-y_1=m(x-x_1)$ to find the equation of a line given a point and its slope.
- Can find the equation of a line through two points by first determining its slope.

B. Start with investigation 3.2

C. After having the students work through the investigation give them practice turning word equations into algebraic and vice versa: "The y coordinate is $\frac{3}{4}$ of the x coordinate plus 2" becomes $y=\frac{3}{4}x+2$.

To help the students internalize the idea that a point and a slope define a line start by having the students graph lines given two points without finding the equation, then have them graph lines given a point and the slope, without finding the equation such as given point (1,3) slope $\frac{5}{2}$ graph the line.



Note that when graphing we can use the slope to find a second point to draw the line to, just as in finding the equation we use a second point to find the slope to form the equation.

D. In the investigation questions 7,8 and 9 are used to show that an alternative to the linear equation exists and can be graphed in the same manner.

In exercise 3-2 the use of parameters for intercepts and slopes is introduced in the latter parts of several questions. The emphasis placed on these parts should be carefully considered in the context of your students ability.

3.3 The Roles of m and b in $y=mx+b$

A. Objectives
- The student can examine an equation in the form $y=mx+b$ and give the slope and y intercept.
- Given the slope and y intercept can form the equation.
- Can take an equation in the form $AX+BY+C=0$ and change it to $y=mx+b$.
- Is introduced to the concept of a family of lines.

B. Start with investigation 3.3

C. The work in this section formalizes ideas that most of the students have already discovered or been taught. Teach the material from the point of view that here are short cuts that can be learned to ease graphing and equation finding.

D. Question 4 developes the slope x intercept form. This idea is not followed up beyond this section and may be omitted by considering (a,o) as a point in the slope point form.

3.4 Parallel and Perpendicular Lines

A. Objectives
- Given the equation of a line the student can give the slope of parallel lines and perpendicular lines and use these to find equations.
- Can determine if lines are parallel or perpendicular by their equations.

B. Start with investigation 3.4. Take up question 1 orally to get started if the students are likely to be confused by the subscripts.

C. If the idea of the slopes of perpendicular lines having product -1 is supplemented by the idea of negative reciprocals it is often easier for students to form the equation of a perpendicular line. $\frac{a}{b} x \frac{-b}{a} = -1$

D. An approach for students having difficulty finding the equations of lines is to have them start the question with the format "Point ________ Slope ________". A point can be derived from a pair of coordinates given to from intercepts written as (a,o) or (o,b). The slope can be given or derived from the slopes of parallel or perpendicular lines, or from coordinates of two given points. The exception to the above is the equation of a line parallel to the y axis which must be learned in the form x=a.

3.5 Applications

A. Objective - The student can apply the skills of analyzing linear equations to problems.

B. Complete the table of values.

X	0	5	10	15			35	20,30,
Y			12		24	36		0,6,18,42

In completing the table what assumption was made?

(That the relation was a direct variation or that the graph passes through the Origin). In a strict sense there can be no wrong answers to completing the table.

C. This section presents again problems involving direct and partial variation so that they may be solved with the help of the geometry of this chapter. On the surface it may appear that y=kx becoming y=mx is the only change however, the information implied by m=slope gives the students a powerful tool.

D. Let the capable students handle this exercise on their own while you consolidate skills with the others.

Test Chapter 3

1. A (3,5), B (-4,2), C (-2,0) are the vertices of a triangle.
   (a) Give the slope of each side.
   (b) What type of triangle is it?
2. Given P(-4,3), Q(1,4) R(11,6)
   (a) Find the slope PQ and PR.
   (b) What do the slopes tell you about the points.
3. Find the equations of the following lines in the form $Ax+By+C=0$
   (a) through (4,-6) slope -4.
   (b) through (6,2) and (-3,0)
   (c) through (-1,3) parallel to $4x+y=6$
4. Find the equations of the following lines in the form $Y=mx+b$
   (a) $6x-5y=2$
   (b) Slope -3/4, y intercept -5
   (c) Through (-4,3) parallel to the y axis
   (d) Through (0,9) perpendicular to $y=3/4x+5$.
5. (a) Graph the following lines on a single set of axes.
   $3x+2y=6$, $y=3x-2$, $y=3/2x-1$, $3x-y+4=0$
   (b) What type of figure is enclosed by the lines
6. (a) The cost of renting a car for the weekend will be $45.00 plus $0.20 per kilometre driven. (a) Write the relation for cost y in terms of the distance driven, x.
   (b) What will the cost be for a trip of 500 km total.

Test Chapter 3 Comments and Answers

Students will require rulers and graph paper.

Answers

1. (a) Slope AB $3/7$, Slope BC -1, Slope AC 1
   (b) The triangle is right angled.
2. (a) Slope $PQ=1/5$ Slope PR $1/5$
   (b) The points are collinear.
3. (a) $4x+y-10=0$, (b) $2x-9y+6=0$ (c) $4x+y+1=0$
4. (a) $y=6/5x-2/5$, (b) $y=-3/4x-5$, (c) $x=3$, (d) $y=-4/3x+9$
5. (b) The figure is a parallelogram.
6. (a) $y=0.2x+45$ or $y=1/5x+45$
   (b) \$145

Review and Preview to Chapter 4

Exercise 1 is a general review of per cent to be used as required. The material in exercises 2 and 3 reviews skills required in this chapter, finding the distance between two points and area of a triangle analytically.

Chapter 4 Distance Between Two Points

This chapter should be covered in from 8 to 10 periods including Review Preview and Testing.

The material in this chapter can be treated at several different levels. The exercises contain questions that lead to the discovery of many geometric properties of triangles. At the other extreme if your students have been having difficulties and the class is behind treat the basic ideas of 4.1 and 4.2 as core.

4.1 Length of a Line Segment

A. Objectives - The student can find the length of the line segment joining any two points given their coordinates.

B. Perform the following subtractions

4-7, 8-(-3), 12-7, -6-5, -4-(-3), -5-(-8), 2.5-3.9, -7.6-12.3, -3.8-4.7, 12.4-(-5.2),

Answers -3, 11, 5, -11, -1, 3, -1.4, -19.9, -8.5, 17.6

C. If the above exercise has been completed use it to introduce the idea of absolute value.

The students should be well practiced in finding Δx and Δy from their work with slope. The topic might be introduced by finding the slope of a line segment between A and B. Take care that the slope has not been reduced if Δy and Δx are to be used for distance.

D. To keep the topic simple answers have been left in radical form. If calculators are in common use you may wish to have the square roots approximated.

4.2 Midpoint for a Line Segment

A. Objectives - The student can find the midpoint of a line segment given the end points.

B. Find the average of the following pairs of numbers

7,5; -4,3; 8, -12; -4, -6; -8, 12; 6.4, 3.8; 4/7, -2.5; -9.8, 7.4; -6.5, -8.2; 5.7, -3.6; Answers 6, -1/2, -2, -5, 2, 5.1, 1.1, 1.2, -7.35, 1.05

C. Once students have recalled how to find the average of two numbers the rest should be straight forward. Have the students do Investigation 4.2, discuss it briefly and let them go on to the exercise.

D. Questions 2 and 3 develop geometric properties of triangles. They can be treated as simple numerical exercises or investigated in detail.

4.3 Collinear Points

A. Objective - The student investigates three diverse mathematical ideas - slope, length, equations - and discovers how they may all be used to reach a single conclusion.

B. Start on the investigation

C. Let the students who have mastered the work to date prepare this unit while you assist the others with fundamental concepts.

D. Question 5 develops the ideas of circumcentre, orthocentre and centroid.

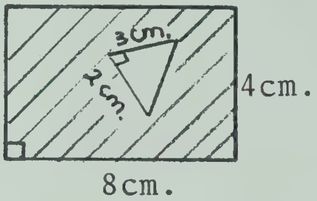
4.4 Area of a Triangle

A. Objectives - The student can determine the area of a triangle given the coordinates of the vertices.

B. Plot the points (5,6) and (-2,0)
Join the points and draw in the run and the rise.
What is the area of the triangle formed? (21 units^2)

Repeat for points (-3,-4) and (2,-8) (10 units^2)

C. This is a good topic to get the student sketching and thinking. An introductory diagram might be as simple as:



Find the shaded area.

(32-3=29cm^2)

D. Question 3 developes the idea that the median of a triangle besects the area and question 4 shows that this method of determining the area can be applied to other figures besides triangles.

Test Chapter 4

1. Find the length of the following line segments. Leave irrational answers in radical form.

(a) A(5,-3), B(5,6) (b) C(-3,2), D(-8,2)

(c) E(6,-5), F(4,7) (d) G(-2,0), H(8,5)

2. Given $\triangle$ ABC, where A(5,4), B(9,-2), C(1,-2). Find the lengths of the sides and determine the type of triangle.

3. Find the midpoint of the following line segments

(a) (6,4), (-4,4) (b) (7,-3), (7,-7)

(c) (-3,9) (4,-3) (d) (1,1) (-3,11)

4. Given the points A(3,-4), B(5,-1), C(6,-8) D(9,5)

(a) Plot the points to determine which three appear to be collinear.

(b) Prove your answer using slope, lengths or equations. (one way only)

5. Find the area of the triangle P(4,6), Q(-2,8), R(0,2)

Test Chapter 4 Comments and Answers

Part of the evaluation for this chapter might include a project poster illustrating one of the geometric properties of triangles developed in the exercises.

Answers

1. (a) 9, (b) 5, (c) $\sqrt{148}$, (d) $\sqrt{125}$
2. The lengths are $\sqrt{52}$, 8, $\sqrt{52}$; isoscles
3. (a) (1,4), (b) (7,-5), (c) (1/2,3), (d) (-1,6)
4. A,B,D are collinear
5. The area is 16 units

Review and Preview to Chapter 5

Exercise 1 reviews arithmetic skills and can be used as calculator practice. Exercises 2 and 3 review ideas of sets and equations that will be required in this chapter.

Chapter 5 Linear Systems

This chapter should be covered in from 15 to 18 periods including Review Preview and testing.

The topics are tightly sequential within two groups; the algebraic solution of linear systems, and the graphical solution of inequations and systems of inequations. Be prepared to treat 5.9 and 5.10 as optional extras if many students have not mastered the preliminary sections.

5.1 Graphing Linear Equations

A. Objectives - To review "quick sketch" methods of graphing linear relations.

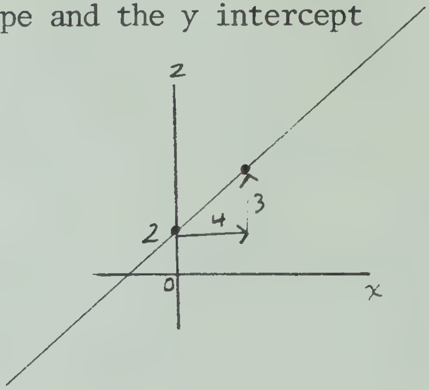
B. $3x + 2y = 12$ if $x = 0$ $y = ?$ (6)
if $y = 0$ $x = ?$ (4)
$5X - 3y = 15$ if $x = 0$ $y = ?$ (-5)
if $y = 0$ $x = ?$ (3)
$6x + 5y = 60$ if $x = 0$ $y = ?$ (12)
if $y = 0$ $x = ?$ (10)

C. The students are being given practice in quickly sketching a line. Little or no written work should be required beyond what is put on the graph. The examples in the text have been expanded for teaching purposes and were not intended to be model solutions for this section.

D. In questions 2 and 3 where values for m and b are suitable, have the lines sketched using the slope and the y intercept directly.

$b = 2$

$m = 3/4$



5.2. Graphical Solution of a Linear System

A. Objectives - The student can solve a linear system graphically.

- Recognizes that linear systems usually have one solution, but that no solution and unlimited solutions are also possible.
- Recognizes the drawbacks to graphical solutions - (lack of accuracy for non-integral solutions).

B. Start directly into the investigation.

C. If your students have been using set notation and operations, discuss the solution of a linear system as the intersection of two sets.

The terms consistent, inconsistent and dependent are useful but need not be memorized.

D. This section should be completed in one period.

The exercise has been kept short for that purpose.

Try to have the work done and taken up by the end of the period to leave the following day open for section 5.3.

5.3 Elimination by Comparison

5.4 Solving Equations in Two Variables by Elimination

A. Objective - The student can solve a system of two linear equations algebraically.

B. Find the smallest term that both of the following terms will divide into (L C M)

1. 3x, x (3x)   2. 5x, 3x (15x)
3. 6x, 2x (6x)   4. 4a, 6a (12a)
5. 8a, 5a (40a)   6. 12a, 8a (24a)

C. The objective is that the student can solve a linear system. For most classes it is appropriate to cover both sections then let the student decide which method to use in a given situation. Weak students will master one method at most and use it, most students will prefer one method and use it, and the best students will be able to select the method most suited to each linear system. If your students seem to be having difficulties with Algebra, select one method and spend twice the time on it, top students can cover the other method independently.

D. Ex. 5-4, Qu. 5 and 6 involve fractions, some students will require guidance since the examples are all integral.

5.5 Problem Solving - Preparation

5.6 Problems Involving Two Unknowns

A. Objectives - the student can analyze a word problem into given information and the question
- can form equations to answer the question
- can solve the resulting equations and answer the questions

B. Start in immediately with Exercise 5.5.

C. Do Qu.1 - Ex. 5.5 orally. With the last several parts, after getting the expression, have the students form an equation - Example (u) If 6 times the price less 4 is $3, what equation is formed? Then have them do Qu. 2 and 3 in written form. If section 5.5 has been covered thoroughly, the emphasis in 5.6 will be on the form that the solution is to take and the required steps.

D. The exercise 5.6 is divided into four parts, each part preceded by an example. It is set up in this fashion to assist students to work through the material at their own pace, calling for assistance as required.

5.7 Graphing Inequations Part I

5.8 Graphing Inequations Part II

A. Objectives - The student can graph linear inequations of the forms $y\ (>, \geqq, <, \leqq)\ mx + b$

B. In each of the following pairs of values determine whether $y < 3x + 4$, $y = 3x + 4$ or $y > 3x + 4$

$(2,{}^{-}4)$, $({}^{-}6,5)$, $(1,7)$, $(5,20)$, $(0,3)$, $({}^{-}2,0)$

C. Section 5.7 concentrates on developing the idea that the line divides the plane into three sets of points defined by $y = mx + b$ $y < mx + b$ and $y > mx + b$. Have the students complete a numerical exercise such as suggested in B, then after separating the ordered pairs into three sets have them draw the line and plot the points to discover the relation.

As the students become comfortable with the ideas of section 5.7, let them move on their own to 5.7.

D. Although it is suggested that relations first be expressed in the form $y = mx + b$, some students are more likely to make errors in this step than in substituting the coordinates of the test point directly into the relation as it is given. These questions occur only in the C exercise, Suggest they try them without altering the forms.

5.9 Intersection and Union of Inequations

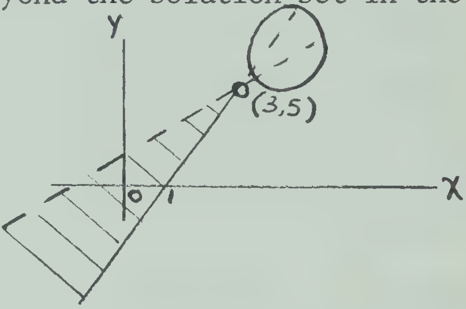
A. Objectives - Students can graph the union or intersection of simple linear inequations.

B. Simplify -

1. $\{3,6,9,12\} \cup \{5,6,7.8.9.10\}$ $\{3,5,6,7,8,9,10,12\}$
2. $\{a,b,c,d\} \cap \{c,d,e,f\}$ $\{c,d\}$
3. $\{\text{even integers}\} \cap \{\text{integers greater than } 5\}$ $\{6,8,10,12, \ldots.\}$
4. $\{\text{Bill, Sue, Cathy}\} \cup \{\text{Cathy, Joe, Jim}\}$ $\{\text{Bill, Sue, Cathy, Joe, Jim}\}$
5. $\{X \in R \mid X < 15\} \cap \{X \in R \mid X > 20\}$ $\emptyset$ or $\{\ \}$

C. Note that there is a printing error in the solution shown to example 1 and in the graphs shown for 5-9, 1(e) and (f). The solid lines should become broken when they pass the point of intersection and go beyond the solution set in the intersection of relations.

Ex. 1



D. It is easier to turn a broken line into a solid line than vice versa. Suggest the students draw in all boundaries broken until they establish which parts are included in the solution set, then draw over the required parts.

5.10 Linear Programming

A. Objectives - To illustrate an interesting application of graphs.

B. Have students go directly to the text.

C. This topic is suitable for use as enrichment by students who have progressed beyond the usual rate or for use by the teacher for presentations as a topic of interest.

Galloping inflation makes liars of us all and it is suggested that the price for the vans in example (1) be doubled.

Chapter 5 Test

1. Solve graphically the system and check by substitution.

   $3x + 4y = 18$ and $y = 2x - 1$

2. Solve the following systems algebraically.

(a) $y = 4x + 8$, $y = -3x + 1$

(b) $y = \frac{1}{2}x - 5$, $x + 4y = -2$

(c) $4x + 9y = 5$, $10x + 6y = 7$

3. Graph the solutions set for $(x,y) \in RxR$

(a) $y > 3x - 2$

(b) $y \leqq \frac{1}{2}x + 3$

(c) $\{(x,y) \leqq RxR \mid y > 3x-2 \cup y \leqq \frac{1}{2}x + 3\}$

4.

(a) One number is three times another. The sum of one half the larger and three times the smaller is 9. Find the numbers.

(b) I have \$5.75 in dimes and quarters and five more dimes than quarters. How many of each can I have?

(c) A traveler flew to a holiday resort at an average ground speed of 300 km/h and drove back at an average speed of 75 km/h. If the total traveling time was 7.5 h, how far was it to the resort?

Test Comments and Answers - Chapter 5

To save students time Qu. 3(c) involves graphing the union of the relations graphed in 3(a) and (b).

The topic of linear programming has not been included in the test. A problem on percentage might be substituted for one of the parts of Qu. 4.

Answers

1. (2,3)

2. (a) (-1,4) (b) (6,-2) (c) $(\frac{1}{2}, 1/3)$

4. (a) The numbers are 6 and 2.

   (b) I have 15 quarters and 20 dimes.

   (c) The resort is 450 km away.

Review and Preview to Chapter 6

The slide rule and the formal method have both fallen victim of the calculator. To maintain some element of human endeavour beyond button pushing it is suggested that the students be required to write an estimate of the answer before calculating. A follow up activity would be to have them calculate their percentage error in estimates from the beginning and end of the exercise.

Chapter 6 Radicals and Exponents

This chapter should be covered in from 12 to 14 periods including the Review Preview and Testing. Although no attempt at formally proving the laws of exponents had been made, formal statements of the laws have been given. This is a first step for students who may go on in technology.

6.1 Mixed and Entire Radicals

A. Objectives - Students can evaluate $\sqrt{a^2}$ as the positive square root i.e. $/a/$
- Recognizes that $\sqrt{a}$, $a < 0$ is undefined.
- Can simplify $\sqrt{a}$, $\sqrt{b}$, $a, b \geq 0$
- Can change the form of radicals, mixed $\Longleftrightarrow$ entire.

B. Express each number as a product of prime factors

1. 16, 36, 125, 196, 625
2. 12, 45, 45, 75, 500

C. These topics are from the grade 10 curriculum, the depth of treatment will depend on your students experience and memory.

The factoring done in the introductory exercise can be used to review finding square roots or simplifying radicals by factoring.

$$\sqrt{16} = \sqrt{(2x2)x(2x2)} \qquad \sqrt{12} = \sqrt{(2x2)x3}$$
$$= 2x2 \qquad = 2\sqrt{3}$$
$$= 4$$

D. Question 7 (optional) could be expanded to include a discussion of absolute value.

6.2 Addition and Subtraction of Radicals

A. Objectives - The student recognizes like radicals
- Can simplify expressions with like radicals
- Can reduce radicals to their simplest form and simplify the expressions.

B. Simplify

1.	$3x+4y+2x-8y$	$(5x-4y)$
2.	$9a-6+4a+5$	$(13a-1)$
3.	$(4x^2+3x+4)-(2x^2-5x+8)$	$(2x^2+8x-4)$
4.	$(2^a+5b)+(4a-5b)-(3a-5b)$	$(3a+5b)$
5.	$2(5x+3)+3(2x-6)$	$(16x-12)$

C. By the distributive property of multiplication over addition $3\times\square + 2\times\square = 5\times\square$ where $\square$ $3x\square$times is replaced by the same number even if we do not know exactly what value that number has.

$\therefore$ $3x+2x=5x$ even if x is unknown similarly

$3\sqrt{2}+2\sqrt{2} = 5\sqrt{2}$ even if 2 is not known exactly.

After the A exercise carry on to expressions with radicals requiring reduction before simplifying.

D. It is not cheating to suggest that when asked to simplify an expression like $4(1)\sqrt{98}-8\sqrt{2}$ the way to start is to evaluate $98 \div 2$ to see if the result is a perfect square.

6.3 Multiplication of Binomial Radicals

A. Objectives - The student can simplify radical expressions involving binomials.

B. Simplify

1. $4(3a+2)$
2. $4x(2x-3)$
3. $-4a(6a-5)$
4. $(3a+2b)(4a-b)$
5. $(3x-2)(2x+4)$
6. $(4x+1)(4x-1)$
7. $(6x-3y)(2x+y)$
8. $(5a-2b)^2$

Answers $12a+8$, $8x^2-12x$, $-24a^2+20a$, $12a^2+5ab-2b^2$

$6x^2-8x-8$, $16x^2-1$, $12x^2-3y^2$, $25a^2-20ab+4b^2$

C. If the analogy to algebra was used in the development of 6.2 then this topic will follow naturally

Expand. $a(3a+2)$ (see above)

Replace a by $\sqrt{2} \Rightarrow 4(3\sqrt{2}+2) =$ __?__

$4x(2x-3)$

Replace x by $\sqrt{5}$, note $\sqrt{5}\cdot\sqrt{5}=5$

$4\sqrt{5}(2\sqrt{5}-3) =$ __?__ etc.

D. Note in questions such as 1(f)

$\sqrt{2}\,(2\sqrt{2}-\sqrt{6}) = 4-\sqrt{12}$ (12 should be reduced)

$= 4-2\sqrt{3}$

This is to ensure that the answer is always left in its simplest form i.e.

$$\sqrt{2}\,(2\sqrt{2} - \sqrt{6} + \sqrt{24}) = 4 - \sqrt{12} + \sqrt{48}$$
$$= 4 - 2\sqrt{3} + 4\sqrt{3}$$
$$= 4 + 2\sqrt{3}$$

6.4 Division of Radicals

A. Objectives - The student can rationalize denominators of radical expressions with monomial and binomial denominators

- Divide radicals of the form $\frac{\sqrt{ab}}{\sqrt{b}}$ $b \neq 0$

B. Find the answer to 4 places of decimal.

(a) 3.4642...÷3

(b) 2÷1.7321...

Note that this is equivalent to evaluating $\frac{2}{\sqrt{3}}$ with and without rationalizing the denominator.

C. This topic is best handled as two separate lessons doing examples 1, 2 and 3 with questions 1,2 and 3, followed by example 4 with question 4.

D. If your students have access to calculators set up a competition between one group that simplifies and then evaluates using calculators and an other group that evaluates directly from the expression using their calculators.

6.5 Basic Laws of Exponents

6.6 Zero and Negative Exponents

A. Objectives - The student learns the vocabulary of powers including the terms base, exponent, power.

- Simplify expressions involving the product of powers to the same base, dividing powers to the same base and raising a power to a power with numeric and algebraic examples.
- Evaluate expressions with zero and negative exponents.

B. Simplify

x^2, x^3, $(2a^3)(6a^4)$, $\frac{x^{12}}{x^6}$, $\frac{16a^3}{4a}$

C. These topics are from the grade 10 curriculum with the examples generalized to illustrate laws. This is a first step towards formal proofs.

D. Question 5 has several parts involving the addition of powers as in example 3(b). This type of question seems to cause problems and deserves a special caution.

6.7 Rational Exponents

A. Objectives - The student can simplify expressions with fractional exponents

- Can interchange fractional exponent notation and radical sign notation.

B. Simplify

1. 3x 1/2 2. 2/3+1/2 3. 1/4+1/2

4. 3/4x4 5. 1/4x(-2/3) 6. 4/5-2/5

C. Fractional exponents are included to complete the domain of the exponential functions presented in the next chapter. It will help lay a foundation to point out at this point that $y=2^x$ for all rational values of x.

D. Caution your students that even though expressions like $27^{5/3}$ may be simplified as either $(27^{1/3})^5$ or $(27^5)^{1/3}$ there is considerable advantage in doing it the first way.

Test Chapter 6

1. Evaluate

(a) $\dfrac{\sqrt{36} + 2\sqrt{25}}{3\sqrt{4}}$ (b) $3\sqrt{6} \times 2\sqrt{6}$

2. Express as mixed radicals

(a) $\sqrt{40}$ (b) $\sqrt{63}$

3. Express as entire radicals

(a) $4\sqrt{5}$ (b) $6\sqrt{10}$

4. Simplify

(a) $2\sqrt{3} + 4\sqrt{2} + 5\sqrt{3} - 6\sqrt{2}$

(b) $\sqrt{5} - \sqrt{12} + \sqrt{45} + \sqrt{75}$

(c) $2\sqrt{28} - \sqrt{175} + 3\sqrt{72}$

5. Simplify

(a) $3\sqrt{2}\,(2\sqrt{2} - \sqrt{3})$ (b) $(3\sqrt{5} - 2\sqrt{7})(3\sqrt{5} + 2\sqrt{7})$

(c) $(2\sqrt{6} + \sqrt{7})(3\sqrt{6} - 2\sqrt{7})$ (d) $\dfrac{3\sqrt{15}}{12\sqrt{5}}$

(e) $\dfrac{5}{\sqrt{3} + 2\sqrt{2}}$

6. Simplify

(a) 5^{-2} (b) $16^{1/2}$ (c) $27^{2/3}$ (d) $(-2)^{-3}$

Test Chapter 6 Comments and Answers

The test has been kept strickly numerical and straight forward.

Answers

1. (a) $8/3$ (b) 36
2. (a) $2\sqrt{10}$ (b) $3\sqrt{7}$
3. (a) $4\sqrt{80}$ (b) $6\sqrt{360}$
4. (a) $7\sqrt{3} - 2\sqrt{2}$, (b) $4\sqrt{5} + 3\sqrt{3}$, (c) $-\sqrt{7} + 6\sqrt{2}$
5. (a) $12 - 3\sqrt{6}$ (b) 17 (c) $22 - \sqrt{42}$ (d) $\frac{\sqrt{3}}{4}$

   (e) $-\sqrt{3} + 2\sqrt{2}$
6. (a) $1/25$ (b) 4 (c) 9 (d) $-1/8$

Review and Preview Chapter 7

Exercises 1 and 2 review skills that will be required when working with the exponential functions in the next chapter.

Chapter 7 The Exponential Function and Logarithms

This chapter should be covered in 5* to 15 periods including Review Preview and Testing.

* As a general education all students should cover sections 1, 2 and 3. The world is subjected to many conditions which vary exponentially. Hence the 5 period minimum. The remaining sections on calculations using exponential and logarithmic functions are more limited in their application and should be assigned time according to the needs of your students.

7.1 Things Which Grow Exponentially

A. Objective - The student is exposed to the ideas of rapid growth associated with exponential functions.

B. Use the table in the margin of P. 126 to make up a table of powers of 4. ($4^0=1$, $4^1=4$, $4^2=16$, ... , $4^6=4096$)

C. This section is designed to get the students thinking and discussing. It should be treated informally.

D. Consistent with the above, you may wish to solve these questions through discussion with the class.

7.2 Graphs of Exponential Functions I

7.3 Graphs of Exponential Functions II

A. Objectives - Through graphing the student investigates the rapid growth of the exponential function
- Notes the effect of changing the base
- Practices graphing and interpolating skills.

B. Start directly into the investigation.

C. Investigations 7.2 and 7.3 are suitable for a group approach with the graphs prepared on experience chart paper with felt markers. The best can then be posted about the room for reference.

D. Exercise 7-2 is a graph reading exercise and can be done orally unless your students need practice. The questions in exercise 7-3 are included for interest and are suitable for enrichment for the groups first finished the investigations.

7.4 Calculations Using the Graph of the Exponential Function

7.5 Computations Using Exponential Tables

7.6 Logarithms

7.7 Multiplication Using Logarithms

7.8 Division Using Logarithms

7.9 Powers and Roots Using Logarithms

A. Objectives - The student can convert between exponential and logarithmic notation.

- The student can use the three laws of logarithms $\log AB = \log A + \log B$

$$\log A/B = \log A - \log B$$

$$\log A^n = n \log A$$

B. Before entering these sections of the work the teacher should carefully assess the needs of the students. In the calculator-computer age no one needs logarithms for simple calculations. Students who may enter technology courses will need to know and apply these laws in equations and using them for calculation is one way to come to understand them.

The fact that scientific calculators have log functions built-in demonstrates this need.

The use of the exponential function for calculation is an easy approach to the use of logs and illustrates the inverse relationship between the functions.

The use of a scientific calculator with a log function to replace the tables and then speed the numerical calculations allows an approach to the topic that can concentrate on the development of mathematical ideas.

Test Chapter 7

1. Complete the following table and use the coordinates to sketch the graphs on the same set of axes.

X	-2	-1	0	1	3
$Y=2^x$					
$y=3^x$					
$y=(\frac{1}{2})^2$					

2. A ball rebounds to 75% of its previous height. If it is dropped from a height of 1600 mm how high will it rebound after hitting the floor for the third time?

3. (a) Change to logarithmic notation.

   i) $2^3=8$ ii) $4^7=x$ iii) $a^y=x$

   (b) Change to exponential notation.

   i) log 16=4 ii) log 9=2 iii) log 1000=3

4. Use tables to find:

   a) log 46.32 b) log 0.921

   c) $10^{3.74}$ d) $10^{0.0439}$

5. Evaluate using logarithms

   (a) 48.65^{10} (b) $\sqrt[3]{0.0965}$

Test Chapter 7 Comments and Answers

Evaluation for this chapter could include a graphing project. In the test, questions on basic operations using logarithms have not been included. They may be added at the teacher's discretion or graded as assignments from the exercises.

Answers

1.

X	-2	-1	0	1	3
$y-2^x$	1/4	1/2	1	2	8
$y=3^x$	1/9	1/3	1	3	27
$y=(\frac{1}{2})^x$	4	2	1	1/2	1/8

2. 675 mm

3 (a) (i) $\log_2 8=3$ (ii) $\log_4 x=7$ (iii) $\log_a x=y$

(b) (i) $2^4=16$ (ii) $a^2=9$ (iii) $10^3=1000$

4. (a) 1.6658 (b) $-1+0.9643$ (c) 3.750×10^3 (d) 1.106

5. (a) 7.412×10^{16} (b) 4.586×10^{-1}

Review and Preview to Chapter 8

This exercise reviews ideas of angles in triangles and properties of similar triangles that are essential for the coming work. It also reviews some ideas in solution of triangles that are covered again in more detail in the coming chapter. If your students can handle questions 5 and 6 without help then be prepared to move quickly over section 8.9.

Chapter 8 Trigonometry

This chapter should be covered in from 15 to 20 periods including the Review Preview and Testing. The lower figure omits the work on obtuse angles and oblique angled triangles in sections 8.6, 8.7, 8.12 and 8.13. These topics lend themselves to enrichment for the mathematically able and those intending to enter courses in technology.

8.1 Angles in Standard Position

8.2 Sine, Cosine and Tangent

A. Objectives - The student can draw any positive or negative angle as an angle in the standard position.

- Use the pythagorean relationship to calculate the missing side in the right triangle formed by the axes and the terminal arm.
- Given the coordinates of a point on the terminal arm calculate the sine, cosine and tangent of any angle.
- Be able to find two positive, first rotation solutions given a trig. ratio for an angle.

B. 1. How many degrees does the minute hand of a clock turn through in 20 min, 30 min, 45 min, 60 min, 100 min, 180 min, 12 h, 1 day, 1 week?

2. In one year how many degrees does

a) the hour hand turn through?

b) the minute hand?

c) the second hand?

Use 3 figures and standard notation

Answer 1. 120^0, 180^0, 270^0, 360^0, 600^0, 1080^0, 8640^0, 60480^0, 2.(a) 7.30×10^2, (b) 8.76×10^3, (c) 5.26×10^5

C. In this section the definitions of angle and the basic trig. ratios are expanded from acute angles to cover angles of all magnitudes both positive and negative, though for practical purposes the magnitudes are restricted.

Radical signs are used to simplify the computation it would be helpful to future work to show that these expressions can be shown in decimal form.

D. Exercise 8-1 question 6 lends itself to oral presentation. Exercise 8-2 question 6 and 7 develop some general trig. identities. The ideas can be extended for enrichment. See the suggestions for section 8.3 before taking up question 2.

8.3 A. Objective - The students learn the definitions and can apply skills of the previous sections to the inverse trig. ratios.

B. State the reciprocals of: 1/2, 4/5, 9 / 7, 1 1/3, 0.6, 0.43, 2.75.

Answers (2, 5/4, 7/9, 3/4, 1.6, 2.33, 0.36)

C. This topic can be taught as an extension to exercise 8-2 question 2. When this work is taken up on the board give the definitions of the reciprocal ratios and extend the solutions, do 8-3 question 1 orally and have the students carry on with this exercise.

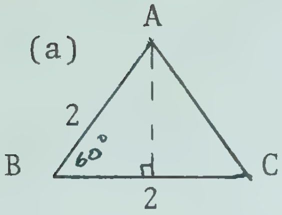
D. This exercise parallels exercise 8-2.

8.4 Special Cases

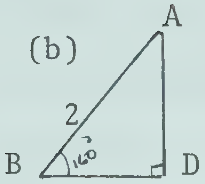
A. Objectives - The student can sketch a 30^0-60^0-90^0 triangle and a 45^0 - 45^0 - 90^0 triangle and derive the trig. ratios of the boundary angle positions (0^0, 90^0, 180^0, 270^0, 360^0) and determine the trig. ratios from the coordinates of points on the terminal arm.

B. Solve the Triangles

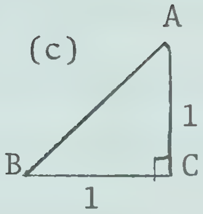
(a)



(b)



(c)



C. Encourage the students to memorize the triangles $1, 1, \sqrt{2}$ and $1, 2, \sqrt{3}$ and then to memorize a table of values of the ratios.

Extend the lesson to include ratios of 0^0 and 360^0 though they are not included in the example.

D. Question 6 illustrates the idea that $\sin A + \sin B \neq \sin (A+B)$ as an identity.

8.5 The Graphs of Sin θ, Cos θ and Tan θ

8.6 The Sign of Sin θ, Cos θ and Tan θ

A. Objectives
- The student can sketch the graphs of the sine, cosine and tangent functions.
- The student recognizes that these are functions and can find similarities between these graphs and the graphs of linear functions and exponential functions.
- Can use the CAST rule to determine the sign of the ratio.

B. Start directly into the Exercise 8-5.

C. The skills of being able to quickly sketch the graph of a function, hitting important intercepts and maximum or minimum values is a very valuable one. It is not intended that the students should draw extremely accurate, detailed graphs.

Carry on and cover 8.6 orally.

D. Questions 6, 7 involve new ideas.

Question 6 can lead the idea of reflecting the graph of $y=\sin\theta$ in the y axis.

Question 7 leads to the idea of adding ordinates from the graph. It might be introduced by having the students derive the graph of $y = 2x+4$ from the graphs of $y=2x$ and $y=4$.

In exercise 8-6 some students may prefer to rely on a sketch graph rather than the CAST Rule. They should be encouraged.

8.7 The Use of Trigonometric Tables

A. Objectives
- The student can find any trig. ratio of angles from 0^0 to 360^0.
- Can find two first rotation angles given the ratio.

B.1. Write the following angles in the form $180^0-\theta$ where θ is acute. 95^0, 107^0, 170^0, 130^0, 156^0.

2. Write the following angles in the form $180^0+\theta$ where θ is acute. 190^0, 210^0, 256^0, 269^0, 217^0.

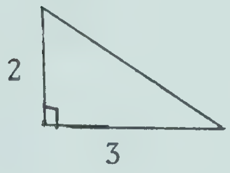
3. Write the following angles in the form $360^0-\theta$ where θ is acute 300^0, 290^0, 315^0, 345^0, 359^0

C. In the sections on sine law and cosine law ratios of obtuse angles are required. Beyond that this topic is included for completeness. Consider the needs of your students when deciding on the depth of coverage.

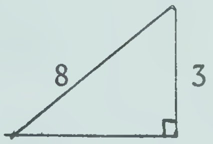
8.8 Trigonometric Ratios of the Acute Angles of a Right Triangle.

A. Objectives - The student can determine trig. ratios of angles in terms of the sides of a right triangle as opposed to the angle being in the standard position.

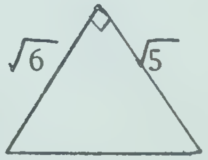
B. Find the length of the third side. Leave irrational numbers in radical form.



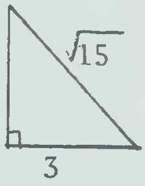
($\sqrt{13}$)



($\sqrt{55}$)



($\sqrt{11}$)

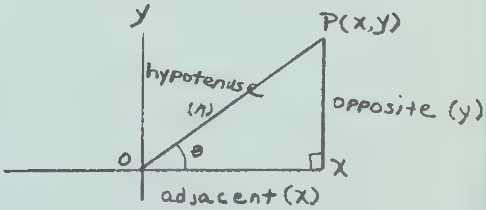


($\sqrt{6}$)

C. If students have difficulty making the transition to triangles a diagram superimposing x, y and r on the triangle may help. The danger is that they use the same diagram for both angles in the triangle.

A large number of students often do not recognize the word adjacent as meaning beside.

D. The A part of this exercise can be handled orally. Question 9 develops two identities in general terms and can be extended to specific cases.



8.9 Solving Right Triangles I

8.10 Solving Right Triangles II

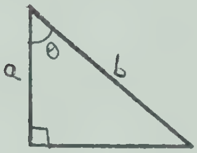
A. Objectives - The student understands the meaning of the expression "to solve a triangle".

- The student can solve right triangles using primary trig. ratios.

- The student can solve right triangles using reciprocal trig. ratios.

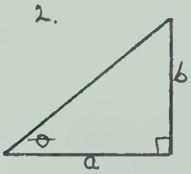
B. Write two equations involving the marked angle and the marked sides.

1.



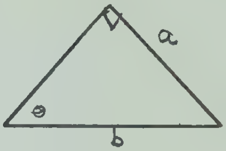
$\begin{pmatrix}\cos\theta=a/b\\ \sec\theta=b/a\end{pmatrix}$

2.



$\begin{pmatrix}\tan\theta=b/a\\ \cot\theta=a/b\end{pmatrix}$

3.



$\begin{pmatrix}\sin\theta=a/b\\ \sec\theta=b/a\end{pmatrix}$

C. Section 8.9 reviews the work of grade 10 and section 8.10 extends it to the use of reciprocal ratios.

An aid to help the students pick a ratio is to use the equation $\frac{\text{unknown side}}{\text{known side}}$ = trig ratio of known angle.

Length of unknown side = Length of known side x trig. ratio of known angle.

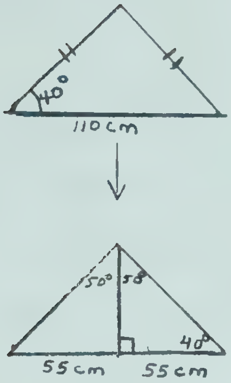
This is covered in 8.11 but can be used with benefit in these sections.

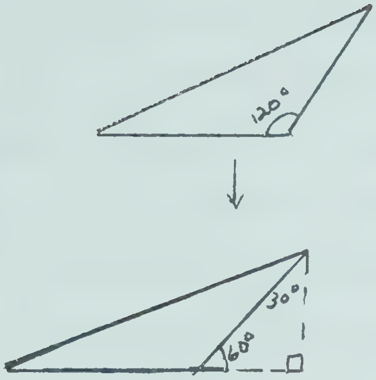
D. The exercises are straight numerical. If you covered the review preview in detail, exercise 8.9 will be review.

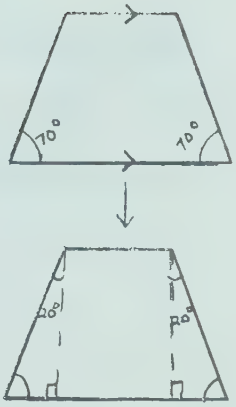
8.11 Applications of Trigonometry

A. Objectives - The student can apply the skills of solving triangles to problem situations.

B. Add lines to the diagrams to make right angled triangles. Mark on additional information that will be known from geometry.







C. The difficulty in this section is usually one of separating out right angled triangles from the diagram or drawing lines to create useful right angled triangles. Encourage the use of coloured pen to outline the triangle being solved.

D. The exercise contains a wide selection of simple survey and shop problems. It is not intended that they all be covered. One option is to give the students an assignment requiring the solution for a set number of problems of their own choosing.

8.12 The Law of Sines

8.13 The Law of Cosines

A. Objectives - The students can solve oblique angled triangles.

B. Express as a ratio of an acute angle.

Sin 112° (sin 68°) cos 165° (-cos 15°)

Sin 94° (sin 86°) cos 115° (-cos 65°)

Sin 173° (sin 7°) cos 94° (-cos 86°)

C. Coverage of these sections should reflect the needs of your students. Proofs of the formulae are included for completeness. It is not intended that the students be required to reproduce them.

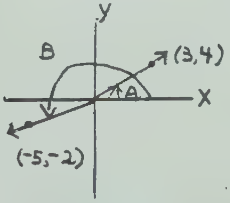
An interest catching method of coverage is to undertake a practical survey problem using a sighting compass or plane table with alidade to measure the angles if a transit is not available, then give the students the necessary formulae to solve the problem.

These formulae are required for the calculation of resultant forces, sec 10.4. The topic might be deferred and taken at that time.

D. The exercises sample the various possible combinations of sides and angles.

Test Chapter 8

1. Give the six trig. ratios of angle A and angle B.



2. Evaluate the following expressions. Give exact values using radical signs if required.

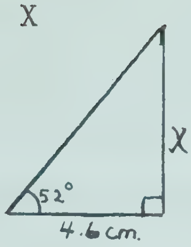
a) $\sin 30^0$ b) $\cos 45^0$ c) $\tan 90^0$

d) $\cos 30^0 \cdot \tan 60^0 + \sin 30^0 \cdot \tan 45^0$

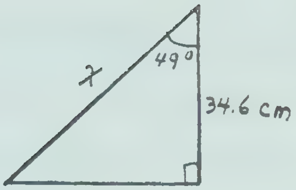
3. Sketch the graph of y=cos x by marking the maximum, minimum and zero points for the domain 0^0 to 360^0 and join them with a smooth curve.

4. Solve for X

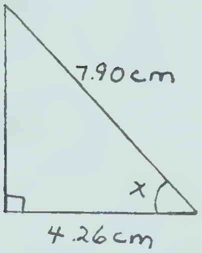
(a)



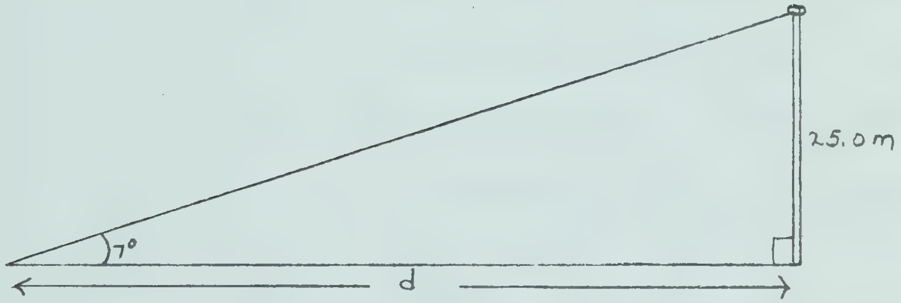
(b)



(c)

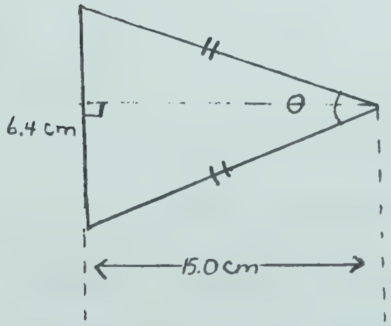


5.



The pole is known to be 25 mm tall. If it subtends an angle of 7° how far away is it?

6.



Find the angle θ at the edge of the wedge.

Test Chapter 8 Comments and Answers

With the exception of question 1 the work on ratios of obtuse angles and solving oblique angled triangles has not been covered on this test.

Answers

1. $\sin A = 4/5$ $\cos A = 3/5$ $\tan A = 4/3$

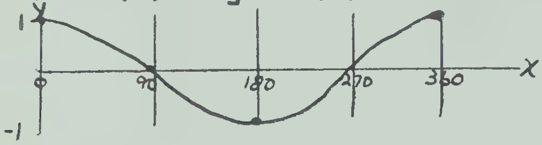
   $\csc A = 5/4$ $\sec A = 5/3$ $\cot A = 3/4$

   $\sin B = -2/\sqrt{29}$ $\cos B = -5/\sqrt{29}$ $\tan B = 2/5$

   $\csc B = -\sqrt{29}/2$ $\sec B = -\sqrt{29}/5$ $\cot B = 5/2$

2. a) 1/2 (b) $1/\sqrt{2}$ (c) undefined (d) 2

3.



4. a) 5.9 cm b) 52.7 cm c) 57°

5. 203.6 cm

6. 25° (24.6°)

Review and Preview to Chapter 9

Exercise 1 reviews basic properties of angles in triangles.

Exercise 2 reviews Pythagorean theorem with example of 3-4-5, 5-12-13, 8-15-17 triangles and their multiples. If the students are watching for these number patterns they can save themselves time and mathematically the search for patterns is a worthwhile exercise.

These topics both have application to the chapter on vectors that follows.

Chapter 9 Vectors in Two Dimensions

This chapter should be covered in from 12 to 15 periods including Review Preview and Testing. Sections 9.7 and 9.9 may be treated as optional.

Square brackets $[a,b]$ have been used to differentiate vectors from coordinates (x,y) and arrows ($\overrightarrow{AB}$) to differentiate vectors from line segments (AB).

9.1 Displacements

9.2 Vectors

A. Objectives

- The student learns to use the terms - displacement, initial point, terminal point, vector, geometric vector, algebraic vector, components.
- Can express a vector as a directed line segment on an ordered pair given the coordinates of the initial point and the terminal point.

B. Plot the following pairs of points and join them. What do you notice about each line segment.

A(3,2), B(7,-1); C(-3,5), D(1,2)

E(0,3), F(4,0) ; G(-7,-2), H(-3,-5)

(Equal and parallel)

C. Students may be familiar with the idea of geometric vectors from grade 10.

Some will notice that we are calculating the run and the rise between the two points. For those who have difficulty let them consider travelling from A to B in a city where we must use the streets and avenues.

D. Stress the sketch diagram for these exercises. A good habit to foster is to have the whole solution done on quad ruled paper.

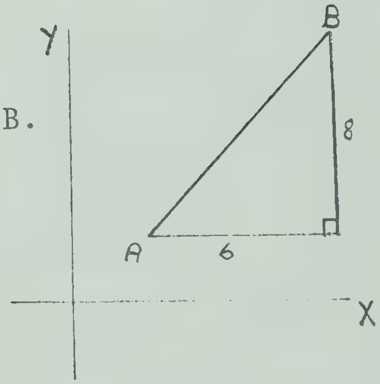
9.3 Equal Vectors

A. Objectives - The student can identify equal vectors.

- Can give the magnitude of a vector

- Recognizes that $[a,b] = [c,d] \Leftrightarrow a=c$ and $b=d$

B.



Give the slope and length of AB

(4/3, 10 units)

C. If the students have done any work on transformational geometry the idea of a vector being translated and still having the same direction and magnitude should come easily. Otherwise a transperancy with a set of axes and a vector and an overlay showing a vector of equal magnitude that can be moved about will help clarify the concept.

D. Questions 3 to 6 are intended to have intuitive graphical solutions followed by an algebraic verification.

9.4 Addition of Vectors

A. Objectives - The student can add algebraic vectors and illustrate the solution geometrically.

B. Solve $3+x=7$, (4); $5-y=0$, (5); $x+(-4)=0$;
$-3+y=6$, (9) $x+(-3)=4$, (-1); $-6+y=-2$, (4).

C. If the students work through the investigation in groups the faster groups can carry on to the B exercise. After taking up the investigation do the A exercise orally. Take special care to note question 13 of the investigation, the negative of a vector.

D. The exercise is straight forward.

9.5 Multiplication of a Vector By a Scalar

9.6 Subtraction of Vectors

A. Objectives - The student can multiply a vector by a scalar.
- Can subtract vectors.
- Recognizes that multiplication by a scalar affects the magnitude but not the direction.

B. Start class directly on the investigation.

C. Treat subtraction of vectors as adding the opposite, let section 9.6 continue directly from 9.5 as a special case where $[a,b]-[c,d]=[a,b]+(-1)[c,d]$.

D. Note that in each exercise some questions involving algebraic vectors with literal components have been included to illustrate that the normal laws of algebra still apply.

9.7 Vectors With Trigonometry

A. Objectives - The student can define a vector in terms of its magnitude and its direction angle.

- Given the vector [p,q] find the magnitude and the direction angle.
- Given the magnitude m and the direction angle θ, find the algebraic vector.

B. Fill in the following table

θ	0	30°	45°	60°	90°	120°	150°	180°	225°
sin	(0)	(1/2)	$(1/\sqrt{2})$	$(\sqrt{3}/2)$	1	$(\sqrt{3}/2)$	(1/2)	(0)	$(-1/\sqrt{2})$
cos	(1)	$(\sqrt{3}/2)$	$(1/\sqrt{2})$	(1/2)	0	(-1/2)	$(\sqrt{3}/2)$	(-1)	$(-1/\sqrt{2})$
tan	(0)	$(1/\sqrt{3})$	(1)	$(\sqrt{3})$	undef.	$(-\sqrt{3})$	$(-1/\sqrt{3})$	(0)	(1)

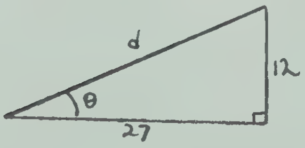
C. The students readiness for this section will depend on their ability to handle trig. ratios of first rotation angles. If you omitted sections 8.6, 8.7 or if your students had great difficulty with them then treat this section as optional.

D. In question 4 the alternate method in part (c) might be drawing geometric vectors and expressing the resultant algebraically.

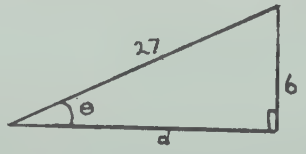
9.8 Problems Solved Using Vectors

A. Objectives - The student can solve problems using vector methods.

B. Solve the triangles for the required information



d=29.5
$\theta = 24^\circ$



d=26.3
$\theta = 13^\circ$

C. Examples 1 and 2 are of general interest since most students are aware of navigational problems in travel. Example 3 deals with alternating current theory and will be of interest to any students in the electricity or electronics option. Note that a west wind blows from the west etc.

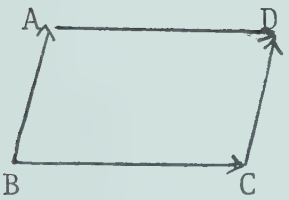
D. Note that questions 1-12 deal with navigation and require basic trigonometry restricted to ratios of acute angles.

9.9 Applications of Vectors to Geometry

A. Objectives - To show the interrelationships and consistency between branches of mathematics.

- The student can solve simple geometric theorems using vectors.

B.



If $\overrightarrow{AB} = \overrightarrow{DC}$ what do you know about AB and CD?

If $\overrightarrow{AD} = \overrightarrow{BC}$ what do you know about AD and BC.

What kind of a figure is ABCD?

(AB = CD, AB//CD
AD = BC, AD//BC
ABCD is // gm)

C. This topic can challenge and inspire the right students. It is suitable for enrichment or optional coverage. Students might be given a choice of preparing a problem package from either 9.8 (applications) or 9.9 (geometry).

D. The exercise deals with some of the more basic Euclidean theorems and can easily be supplemented from any geometry text.

Test Chapter 9

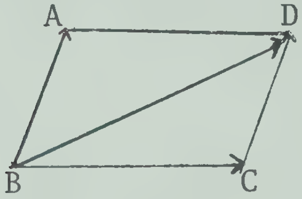
1. Express as an algebraic vector, given A(3,5), B(-2,6), C(0,4)

are three points:

(a) $\overrightarrow{AB}$ (b) $\overrightarrow{CB}$ (c) $3\ \overrightarrow{AC}$

(d) $\overrightarrow{AB} + 3\overrightarrow{AC}$ (e) $\overrightarrow{AB} - \overrightarrow{CB}$

2.



Name a vector (a) equal to $\overrightarrow{BA}$

(b) equal to $-\overrightarrow{BC}$

(c) equal to $\overrightarrow{BA} + \overrightarrow{AD}$

3. Given $\vec{x} = [12,-5]$, $\vec{y} = [-3,4]$, $\vec{z} = [5,1]$,

simplify (a) $\vec{x} + \vec{y} - \vec{z}$

(b) $2\vec{x} + 3\vec{y}$

(c) $/\vec{x}/ + /\vec{y}/$

4. (a) Give the magnitude and direction angle of the vector $[\sqrt{3}, 1]$

(b) A vector has magnitude 5 and direction angle 45°. Give it as an algebraic vector.

5. A motor boat heads directly across a river at 30 km/h. The river has a current of 5 km/h. What is the actual speed of the boat?

Text Chapter 9 Comments and Answers

The test does not require trig. ratios of obtuse angles nor tables. Graph paper would assist. Geometric proof have not been tested.

Answers

1. (a) $[-5,1]$, (b) $[-2,2]$, (c) $[9,-3]$, (d) $[-14,-2]$,
(e) $[-3,-1]$

2. (a) $\overrightarrow{CD}$, (b) $\overrightarrow{DA}$, (c) $\overrightarrow{BD}$

3. (a) $[4,-2]$ (b) $[15,2]$, (c) 18

4. (a) 2, 30°

   (b) $\left[\frac{5\sqrt{2}}{2}, \frac{5\sqrt{2}}{2}\right]$ or $\left[\frac{5}{\sqrt{2}}, \frac{5}{\sqrt{2}}\right]$ or $[3.5, 3.5]$

5. $5\sqrt{37}$ km/h or 30.4 km/h

Review and Preview to Chapter 10

This is a self contained unit inserted to give the student practice in calculating proportional parts.

Chapter 10 Statics Coplanar Forces

This chapter should be completed in from 10 to 12 periods including Review Preview and Testing. Sections 10.4, 10.5 and 10.6 are dependent on the ability to solve oblique angled triangles. The whole chapter is an extension on the work with vectors.

10.1 Units of Force

A. Objectives - The student can define force as mass times acceleration

- Can use the formula F=ma to calculate any missing part.
- Gains an intuitive understanding for the concept of force and the units employed.

B. Draw the following vectors - magnitude 5, upward.

- magnitude 7, 45° down to left
- magnitude 3, to right
- magnitude 10, 45° up to right.

C. Students should be introduced to the destinction between force and mass. A useful illustration is that of the astronaut whose mass remains relatively constant but whose weight (gravitational force) will vary from normal to zero during the trip. For a more detailed study the application of G forces during take off acceleration might also be included.

D. The exercise involves simple vector diagrams. The important difference is the addition of units since we are now dealing with applications.

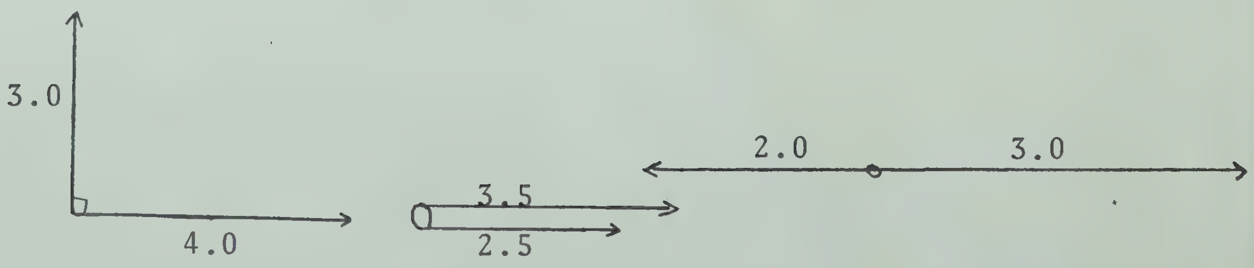
10.2 The Resultant

10.3 The Equilibrant

A. Objectives - The student can use a scale drawing to calculate the resultant of two coplanar forces.

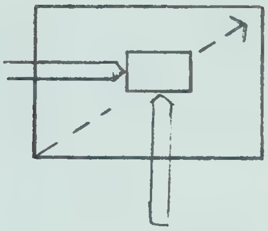
- Can use the resultant to calculate the equilibrant of forces.

B. Find the sum of the given pairs of vectors by scale diagram.



C. The idea of combined effect can easily be demonstrated by having the students move an eraser diagonally across the desk by pushing it with a pen in each hand in directions parallel to the edges of the desk only.

As the students complete exercise 10-2 let them carry on independently to Investigation 10.3 and exercise 10-3.



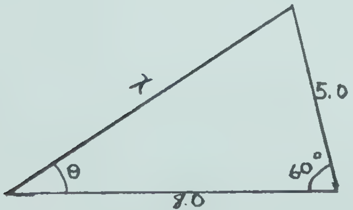
D. Stress the value of careful diagrams. A valuable exercise would be to have one question done to a large scale on chart paper. Have the angle and magnitude measured as accurately as possible. Save the sheet for use in section 10.4 and have the results checked by calculation.

10.4 Finding the Resultant with Trigonometry.

A. Objectives - The student can calculate the magnitude and direction of non-collinear coplanar forces using trigonometry.

- Apply the theory to practical problems

B. Find the required information in the triangles.



$\begin{pmatrix} x = 7.0 \\ \theta = 38^{\circ} \end{pmatrix}$

C. This topic requires that sections 8.12 and 8.13, Law of Sines and Law of Cosines were covered. If not covered previously they could be done at this time if the preliminary topics were done.

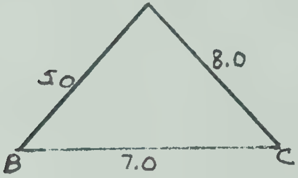
D. Encourage realistic diagrams to act as a check on calculations.

10.5 Equilibrium at a Point

A. Objectives - The student intuitively understands the concept of equilibrium

- The student can arrange three coplanar forces to produce equilibrium or demonstrate the impossibility.

B. Find the angles in the triangle



($\angle A=60^{\circ}$
$\angle B=82^{\circ}$
$\angle C=38^{\circ}$)

- Is it possible to construct a triangle with sides 6 cm, 3 cm and 2 cm? Why?

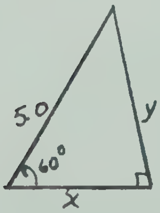
C. A practical demonstration is very helpful with this topic. A piece of peg board with hooks and spring scales calibrated in newtons makes a useful piece of apparatus. Paper taped behind the scales allows vectors to be drawn on the board.

D. Note question 3 gives a right triangle and in question 4 the forces are collinear.

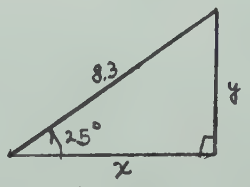
10.6 Resolution of Forces

A. Objectives - The student can resolve a force into two rectangular components.

B. Find the lengths of the sides.



($x = 2.5$
$y = 4.3$)



($x = 7.5$
$y = 3.5$)

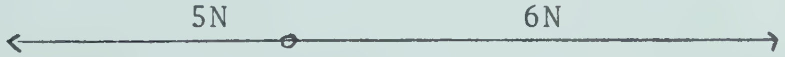
C. This topic may be introduced through algebraic vectors.
Example - What horizontal and vertical vectors add to give the vector [3,4] ? ([3,0] + [0,4])
The mathematics is so simple that they will focus on the idea.
The above example translates into finding the horizontal and vertical components of a 5N force at 53° to the horizontal.

D. A further development from question 2 might be to investigate why it is easier to pull the sleigh (toboggan) than to push it. The upward component of the pull tends to reduce friction. While the downward component of the push tends to increase it.

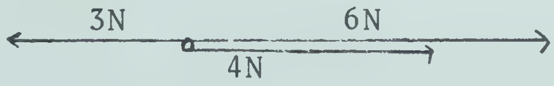
Test Chapter 10

1. What force is required to accelerate a 50 kg rocket at 200 m/s^2?
2. State the resultant of the following system of forces.

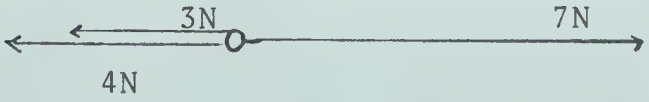
(a)



(b)

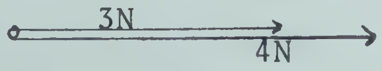


(c)

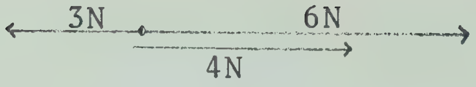


3. A force of 8N makes an angle of 60° to a force of 5N. Draw a scale diagram and find the magnitude of the resultant and the angle that it makes with the 8N force.
4. Find the single force that puts the following systems in equilibrium.

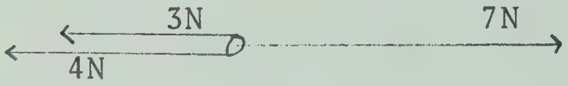
(a)



(b)



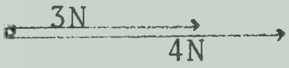
(c)



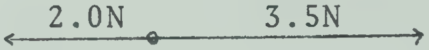
3. A force of 8N makes an angle of 60° to a force of 5N. Draw a scale diagram and find the magnitude of the resultant and the angle that it makes with the 8N force.

4. Find the single force that puts the following systems in equilibrium.

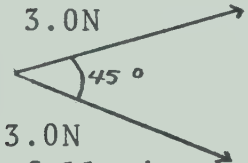
(a)



(b)

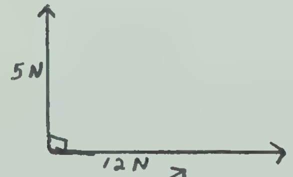


(c)

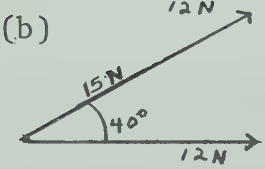


5. Calculate the following resultants given the magnitude and the angle it makes with the larger given force.

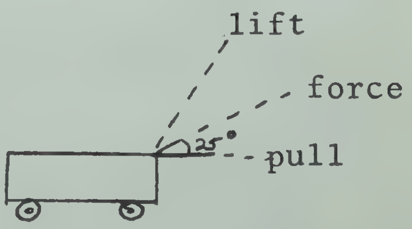
(a)



(b)



6. a) A wagon is pulled with a force of 50N. The force makes an angle of 25° to the horizontal. What component pulls the wagon and what component tends to lift the wagon?



b) Why is the wagon easier to pull with a longer handle?

Test Chapter 10 Comments and Answers

The first 4 questions can be done without the use of trigonometry. For question 5 and 6 tables of trig. values are required. Rulers and protractors are required for question 1-3.

Answers

1. 10 000 N or kg $m/5^2$
2. (a) 1 N to right, (b) 7 N to right, (c) 0 N or equilibrium
3. 11 N, 23°
4. (a) 7 N to left (b) 1.5 N to left (c) 5.5 N at 157 1/2° from either force
5. (a) 13 N at 23° (b) 25.4 N at 18°
6. (a) Pull = 45.3 N, left = 21.1 N. (b) a longer handle increases the pull component and decreases the lift.

Chapter 11 Review and Preview

Chapter 11 covers the analytical geometry of the circle involving equations of circles, equations of tangents and properties of chords and bisectors. This Review and Preview exercise reviews the Pythagorean theorem, properties of a circle and equations of lines - all of which will be required for the coming topics. In exercise 1 question 2 substitute calculator for slide rule.

Chapter 11 The Circle.

This chapter should be covered in from 10 to 12 periods including Review Preview and Testing. It investigates the geometry of the circle through an analytic approach. Several of the exercises are used to expand the ideas of the sections rather than just practice skills.

11.1 Equation of a Circle with Centre the Origin

11.2 Graphs Determined by $x^2+y^2=r^2$

A. Objectives - The student can determine the equation of a circle with centre at the origin given the radius.

- Recognizes that the equation $x^2+y^2=r^2$ divides the plane into three areas:

Points inside the circle $\{(x,y)\in R\times R \mid x^2+y^2<r^2\}$

Points on the circle $\{(x,y)\in R\times R \mid x^2+y^2=r^2\}$

Points outside the circle $\{(x,y)\in R\times R \mid x^2+y^2>r^2\}$

B. On a piece of graph paper construct a circle with centre at the origin passing through the point (5,8). Mark on three other points with integral coefficients that are on the circle. $((\pm 5, \pm 8), (\pm 5, \mp 8))$. What is the radius of the circle?

C. The equation can be developed through the word equation as an intermediate step.

Distance from centre (0,0) to P(x,y) on the circle $= r$ (radius).

As the students complete exercise 11-1 let them work on into investigation 11.2.

D. The exercise stresses the idea that if the coordinates satisfy the equation, the point is on the graph.

11.3 The Equation of a Circle With Centre Not at the Origin

A. Objectives - The student can find the equation of a circle given its centre and radius.

- Given an equation in the form $(x-h)^2 + (y-k)^2 = r^2$, identify it as the equation of a circle and give the centre and radius.

B. Start students directly on the exercise.

C. The development of this section should follow as a logical extension of 11.1. Keep the method of presentation as similar as possible.

Word Statement → word equation → algebraic equation.

D. Questions 1-5 follow the example exactly, questions 6 to 11 require some original thinking.

11.4 Equation of a Tangent To a Circle

A. Objectives - The student can find the equation of a tangent to a circle at a given point on the circle, working from basic principles.

B. Find the slope of a line segment perpendicular to the line through:

(6,2) and (-4,3)	(10)
(4,6) and (4,-3)	(0)
(-2,8) and (5,8)	(undefined)

C. The examples and B exercise are restricted to tangents at a point. The C exercise investigates tangents with a given slope.

D. Question 5 is in parts to lead the student to a solution and this is also applicable to question 6.

11.5 Length of a Tangent To a Circle

A. Objective - The student can find the length of the tangents to a circle from a point outside the circle.

B. Find the distances between the pairs of points

A (-3,4)	P (5,6)	($\sqrt{68}$ or $2\sqrt{17}$)
C (0,2)	D (1,8)	($\sqrt{37}$)
E (5,7)	F (9,6)	($\sqrt{17}$)

C. The students should soon get the idea that two equal tangents can be drawn from an external point. This topic is restricted to development from general principals. The development of the formula could be an enrichment topic.

D. Question 1-6 follow from the example. Questions 7 to 10 require original thinking.

11.6 The Right Bisector of a Chord.

A. Objectives - The student can use the methods of analytical geometry to illustrate properties of circles and solve problems.

B. Find the midpoint of the following line segments

A (4,3)	B (-2,5)	(1,4)
C (0,7)	D (0,-3)	(0,2)
E (-4,6)	F (5,-6)	(½,0)

C. This section involves many ideas and is an exercise in applying the skills developed to date. It could be used as an optional topic for students who have mastered the material while those having difficulty are getting additional help.

D. Ideas developed in the exercise

Question 1 - The right bisector of a chord passes through the centre of a circle

2 - A line drawn from the mid point of a chord to the centre is perpendicular to the chord.

3 - A line drawn perpendicular to a chord through the centre bisects the chord.

4, 5, 6 - Finding the equation of a circle through three given points.

7, 8 - The angle in a semi circle is a right angle.

9 - In two intersecting circles the common chord is perpendicular to the line of centres.

Test Chapter 11

Sketch a diagram for all questions.

1. Find the equations of the following circles.

   (a) centre origin radius 6 units

   (b) centre origin passing through (5,4)

   (c) centre (4,-2) radius 4 units

   (d) centre (-1,5) passing through (4,3)

2. (a) Graph the area defined by

   $$\{(x,y)/\ x^2+y^2 < 36,\ x,\ y \in R\}$$

   (b) Is the point (5,-3) in the area graphed in (a)?

3. Find the equation of the tangent to the circle

   (a) $x^2+y^2=34$ at the point (5,-3)

   (b) $(x+3)^2+(y-2)^2=13$ at the point (0,4)

4. Find the length of the tangent from (8,6) to the circle $(x-5)^2+y^2=9$

5. (2,-5) and (4,7) are the ends of a diameter of a circle. Find the equation of the circle.

Test Chapter 11 Comments and Answers

The students will require rulers, compasses and graph paper. Be generous with the marks for good diagrams.

Answers

1(a) $x^2+y^2=36$, (b) $x^2+y^2=41$, (c) $(x-4)^2+(y+2)^2=16$ (d) $(x+1)^2+(y-5)^2=29$

2(a) Interior of circle, centre origin radius 6.

(b) yes

3(a) $5x-3y=34$, (b) $y=\frac{-3}{2}x+4$

4. 6 units

5. $(x-3)^2+(y-1)^2=37$

Chapter 12 Review and Preview

These exercises review the skills of working with per cents and calculating fractions of a year, operations necessary for the use of the basic formula

$I = PRT$

Estimating by the 6% for 60 day method is useful as a quick check method. One of the most valuable skills a student can learn is to be able to quickly determine if an answer is reasonable.

Chapter 12 Borrowing Money

This chapter should be covered in from 13 - 15 periods including Review Preview and Testing. It covers the basic mathematical ideas that evolve from the calculation of simple interest. Over all the mathematical objectives should be retained as well as the objective of increasing student awareness of the cost of borrowing.

12.1 Financing With Simple Interest

A. Objectives
- The student learns the terms - amount, principal, interest, rate, time.
- Can use the formulae $A=P+I$, $I=Prt$

B. Given I=Prt solve for P, r and t

(P=I/rt, r=I/Pt, t=I/Pr) see the margin p. 267

C. The complete table in Fig. 12-1 "Fractions of a Year" appears on p. 379.

The fast changing economic situation makes it impossible to stay current with interest rates. The material in the text should be supplemented by advertisments and news items giving interest rates being charged on bank loans and charge accounts.

D. Example 12-1 question 10 is in preparation for the work on compound interest which is to come in chapter 13.

12.2 The Cost of Borrowing Money

Effective Annual Rate

12.3 Instalment Buying

A. Objectives

- The student can calculate the real cost of borrowing money or buying on instalments.
- To increase the students awareness of the cost of borrowing.

B. Find the average of the following amounts of money.

$725.30 and $93.50 ($409.40)

$5285.00 and $165.40 ($2725.20)

$1246.50 and $1400 ($630.25)

C. With current laws demanding disclosure of the cost of borrowing the need for borrowers to be able to calculate the cost of borrowing is reduced. However calculating the the rate should increase the awareness of the effect of the rate on the total cost of the purchase. It will also serve to familiarize the students with the terms used in borrowing. For the sake of the mathematical thinking involved the average principal method is to be preferred.

D. The exercises might be supplemented by having the students check the published interest rates for consumer loans. Exercise 12-3, question 11 and 12 are largely algebraic in nature and should be optional.

12.4 Paying Principal Plus Accrued Interest

12.5 Borrowing on a Time Note

A. Objectives - The student is made aware of various methods of loan repayments.
- Learns the terminology and mathematics associated with each.

B. A loan is repaid by monthly payments of \$110, \$108, \$106, \$104, \$102.

How much was borrowed?	(\$500)
How much was repaid?	(\$530)
What was the monthly interest rate?	(2%)

C. Although paying principal plus accured interest is an unusual way to repay a loan it serves to illustrate the effect of reducing principal on the cost of borrowing.

The repayment schedule and the promisory time note are two documents that all borrowers will be required to deal with whether they are buying a house, a car or major appliances. Becoming familiar with the terminology is probably the most valuable objective.

Stress the fact that a promissory note is only as good as the person who gives it and that legal collection costs usually make the process unprofitable. On the other hand borrowers need to protect their credit ratings and should be careful not to borrow beyond their ability to repay.

D. A mimeographed sheet of blank promissory notes will speed the work of exercise 12-5. This is a good project for one of the faster students.

12.6 Discounting Notes

A. Objectives - The student is introduced to the idea and mechanics of discounting a note.
- Can use a time diagram to assist in calculating the times for interest calculations.

B. Find the number of days and fraction of a year between:
06-15 and 08-25, 02-01 and 05-30, 03-19 and 07-21,
07-19 and 02-06 year following
(71d, 0.1945a; 118d, 0.3233a; 124d, 0.3397a, 202d, 0.5534a)

C. Two important ideas to be stressed are the time-money relation, money due in the future is of less value than money due now - just as a debt due now is increased as its payment date is put off into the future; and the idea of graphing the date of borrowing and due date on a time line. The time line will be developed and used in greater detail when studying annuities.

D. Question 4 illustrates the idea that a $100 loan discounted at 8% really only supplies a principal of $92 therefore the interest of \$8 provides an interest rate in excess of 8% (8.7%)

12.7 Graphs of Interest Relations

A. Objectives - The student can apply graphing skills to the relation $I=Prt$ where various pairs of values are kept constant.

- Gain an insight into how the growth of I and P are affected by changes in t and r.

B. Sketch the graph of $y=\frac{3}{4}x$ and $y=12/x$ for positive values of x.

C. Besides reviewing graphing skills this section should help students gain an insight into how the various parameters are related.

A suggested project is to have several graphs drawn on a single set of axes. For example \$1000 invested at 10%, 15% and 20% graph I against t, $0 \leq t \leq 1$.

D. Question 9 introduces amount as a variable to be graphed. Compare the relation $A=(Pr)t+P$ with $y=mx+b$ to help the students find a quick way to sketch the graph.

Chapter 12 Test

1. Complete the table

	I	P	r	t	A
a)	////	\$600	12%	0.5a	////
b)	\$75	\$1000	////	9 mo.	////
c)	\$225	////	18%	3 mo.	////
d)	\$1440	////	24%	////	\$13 440

2. Sue Brown bought a car for $2000 cash and 36 monthly payments of $225. The cash price of the car including tax and all extras was $7 720.00 What interest rate was she being charged?

3. (a) A 30 day note for $600 is made on 15 Mar. What is the due date?

   (b) If it bears interest at 20% what is the maturity value?

   (c) If it is discounted at 24% on 20 Mar. what are the proceeds?

4. a) Complete the table for an investment of $2 000 at 12%

t(a)	0	2	5
I($)			

   b) Graph the relation for 0 to 8 years.

   c) From the graph find I for 1.5a and 7.5a.

Test Chapter 12 Answers and Comments

The evaluation on this chapter might include a project on consumer credit. A comparison of rates for bank loans, credit union loans and credit card accounts would be a suggestion.

For this test the students require graph paper.

Answers

1(a) $36, $636 (b) 10%, $1075, (c) $000, $5225, (d) $12 000, 015

2. 26.7%

3.(a) .17 Apr., (b) $610.85 (c) $599.60

4.(a) $0, $480, $1200 (c) $360, $1800

Review and Preview to Chapter 13

This exercise practices the ideas of simple interest that are required for the work in compound interest to follow. The use of calculators is recommended.

Chapter 13 Compound Interest

This chapter should be completed in from 13 to 15 periods including Review

Preview and Testing.

It covers the basic ideas of principal and amount invested at compound rates. Again, for the sake of realism check current rates for additional examples. If time is running short sections 13.4 and 13.5 may be omitted with least effect.

13.1 Growth of Money Under Compound Interest

A. Objectives - The student is introduced to the basic ideas of compound interest
- Learns the terms compound, conversion period

B. Write as a decimal (a) 6%, 12%, 20%, 24%
(b) 1+8%, 1+15½%, 1+20%, 1+25% (c) ½ of 12%, ¼ of 10%, ½ of 24½%, ¼ of 30%

(answers (a) 0.06, 0.12, 0.20, 0.24
(b) 1.08, 1.155, 1.20, 1.25
(c) 0.06, 0.25, 0.1225, 0.075)

C. After discussing examples 1 and 2 on p.282, 283 let the students work in pairs on the investigation 13.1. If the students encounter difficulty the exercise can be taken up orally while the students write up solutions in their notes. The key question is #5 on p. 284 and example 3.

D. Note exercise 13-1 question 1 should read Rate per annum (r). These questions lead to the use of tables. Without calculators the multiplication becomes tedious. Adjust the assignment according to their availability.

13.2 Compound Interest Tables

A. Objectives - The student can use tables to find the amount of an investment given the principal, time, interest rate and conversion period.

B. Find the number of conversion periods in:

4a quarterly (16), 6a semi annually (12)
2½a monthly (30), 42 mo quarterly (14)
15a semi annually (30), 8½a quarterly (34)

C. If 13.1 has been mastered this section should follow directly from it. If some students are having difficulty let the others work on in this section while you work with small groups.

D. Tables of values for $(1+i)^m$ are on p.384. A piece of tape folded to make a tab on the page speeds the work of the exercise. Students with calculators with a y^x function should be shown how to eliminate the need for tables.

13.3 Problems Involving Amount

A. Objectives - The student can use a time diagram to illustrate problems.

- The skill of finding amounts can be applied to problem situations.

B. Draw a time line for 3 years starting at "Now". Write numbers at the end of each year and draw lines at the half year intervals.

C. Stress the value of a neat diagram containing the given information. Once this skill is mastered the solutions fall neatly into place.

D. One of the most frequent sources of error is the calculation of the amount of time between dates (question 8, 9, 10). A carefully drawn time line solves the problem.

13.4 Fractional Parts of a Time Period

A. Objectives - The student can find amounts involving fractional time periods.

B. Find the amount
(a) \$600 at 12%/a for 2 mo. (\$612)
(b) \$200 at 18%/a for 12 d. (\$201.18)
(c) \$500 at 24%/a for 4 weeks (\$511.51)

C. The skills involved here are just those of sequencing two previously covered ideas. It might be approached by going back to one of the parts of question 1 in exercise 13-3 and extending it. For example, after reviewing the solution to

(a) \$400 in 5a at 8% compounded annually extend the time to 5a 4 mo, or 5a 30d.

Project suggestion - have class members check advantages to daily interest bank accounts. How do they differ from regular savings accounts.

D. The exercise follows the examples closely.

13.5 Problems Involving Rate and Time

A. Objectives - The students can use the tables to gain approximations of rate or time required to achieve a given amount.

B. Solve

$\frac{0.0365}{0.0948} = \frac{x}{1}$ (0.385) $\frac{0.1495}{0.2134} = \frac{x}{0.5}$ (0.35)

C. This topic can be approached at two levels. For most students the ideas involved in approximating the rate or time directly from the tables without a calculated interpolation will be adequate. The ideas of interpolation occur later in chapter 15, calculation of percentiles.

D. In question 8 it is necessary to calculate the growth factor for \$1000 to become \$2500. This may cause some difficulty without preparation.

13.6 Present Value of an Amount

A. Objectives - The student is introduced to the idea of present value.

- Can calculate the present value of an amount given the time, interest rate and conversion period.

B. Solve for P.

(a) $\$4154 = P\ (1.34)$ (b) $\$5588 = P\ (1.016)$

(c) $8 + P\ (3)^{-2}$ (d) $12 = P\ (2)^{-4}$

answers (a) \$3100 (b) \$5500 (c) 72 (d) 192

C. This topic can be tied back to discounting a note. Money owed in the future is not as valuable in the present. A principal invested now will increase in amount though perhaps not in value. Discuss the effect of inflation. Take a simple example such as "What principal invested at 16% compounded semi annually for 5a will amount to \$5000?" Have half the class solve $P = \frac{5000}{(1.08)^{10}}$ using tables p. 384. While the rest solve $P = 5000 \times \frac{1}{(1.08)^{10}}$ using tables p. 385. Discuss relative merits of each form using pencil and paper or using calculator. Use a calculator with y^x function to show 1.08^{-10}.

D. Questions 1 and 2 involve single amounts, questions 3 to 10 involve sums or comparisons of two or more amounts.

Test Chapter 13

1. Find the following amounts
   (a) \$5000 invested at 8% compounded annually for 5a.
   (b) \$3000 invested at 16% compounded quarterly for 6a.
2. Find the following principals.
   (a) Invested at 12% compounded semi annually amounts to \$8000 in 4a.
   (b) Invested at 24% compounded monthly amounts to \$800 in 3a.
3. At 20% compounded quarterly, for how many complete conversion periods must money be invested to double?
4. How much money must be invested now at 16% compounded semi annually to pay a mortgage of \$40 000 due in 20 years.
5. Find the amount of \$525 invested at 12% compounded semi annually for 3a 8 mos.

Test Chapter 13 Comments and Answers

The amount of time required for this test will vary depending on whether calculators are allowed. Tables will be required or calculators with a y^x function.

Note that question 3 is taken from tables to the next complete conversion period. If you have spent time on interpolation this question should be amended.

Answers

1. (a) \$7346.64 (b) \$7689.91
2. (a) \$5019.30 (c) \$392.18
3. 15 conversion periods. 4. \$1841.24 5. \$805.19

Review and Preview to Chapter 14

This exercise reviews basic operations with amounts of money. These skills are required in the following chapter.

Chapter 14 Buying and Selling, Profit and Loss

This chapter should be covered in from 15 to 17 periods including Review, Preview and Testing. It covers the basic ideas in calculating profit, loss, net price after discounts, margin and markup.

The consistent use of a calculator is recommended since examples and exercises should use realistic figures involving dollars, cents and fractional percentages.

14.1 Profit and Loss

14.2 Inventory and Net Profit

A. Objectives - The student learns the terms profit, loss, gross, net, and inventory.

- Can use the three basic equations covered in these sections to calculate gross profit or loss, net profit or loss and cost of goods sold.

B. If 500 men's ties are purchased at $4.25 each and sold at $6.50 what profit is made on each tie? On the total purchase? ($2.25, $1,125.00

C. A discussion of the mechanics of a paper route is a good introduction to the topic. Consider such items as the number of customers on the route, the cost of papers, the amount collected from a customer for weekly delivery, what happens when customers don't pay, how does the carrier get paid for "office pay" customers, tips, the costs of delivering the paper, bike repairs, etc.

D. Exercise 14-1 Qu. 6 leads to the idea that money tied up in inventory can be compared to money invested and therefore the length of time that it takes stock to "turn over" affects the rate of return on the investment.

14.3 Profit and Loss Statement

A. Objectives - The student can draw up a statement of profit and loss for a simple business.

- Start to appreciate the value of keeping accurate business records.

B. Express as a percent. The second number in each case represents 100%.

$479 of $936, $220 of $1,040, (51%, 21%)

$250 of $225, $10,590 of $250,000 (111%, 4%)

C. After looking at the statement of profit and loss shown on page 305, put the following on the board.

Net Sales	$585 000	100%
Cost of Goods Sold	351 000	60%
Gross Profit	234 000	40%
Operating Expenses	204 750	35%
Net Profit	$29 250	5%

Open a discussion into which business made the most money and which business appears to be in the healthier state. If you were hired to operate the second business where would you look to improve the profit?

D. The exercise follows directly from the examples.

14.4 Margin and Markup

14.5 Converting Margin and Rate of Markup

A. Objectives - The student can calculate margin and rate of margin from cost price and selling price.

- Calculate markup and rate of markup from cost price and selling price.
- Calculate selling price from cost price and rate of markup or rate of margin.

B. Simplify

$$\frac{4.7 - 2.3}{2.3} \times 100\% \quad (104\%) \qquad \frac{7.4 - 5.9}{5.9} \times 100\% \quad (25\%)$$

Solve

$x = 6\ (9+x)$ $\quad$ $(x = \frac{-54}{5})$

$x = 0.3\ (5+x)$ $\quad$ $(x=2.14)$

C. The difficulty in this section is usually with the conversion of rate of margin to rate of markup. Consider supplying the class with a graph of margin against markup for section 14.5 if the work of 14.4 has not been mastered.

D. In exercise 14-4 Questions 7 and 8 reinforce the stores planning and setting rates of margin on the basis of estimated expenses and sales. Discuss how a store manager would estimate these amounts.

In exercise 14-5 the graph drawn in question 1 is used for the solutions in the rest of the exercises. (See the note in C)

14.6 Trade Discounts

14.7 Equivalent Single Discount

A. Objectives - The student can calculate the discount and net price given the rate of discount.

- Calculate the single rate of discount that replaces a series of discounts.

B. Find the following amounts.

12% of $46.72 ($5.61)   9.5% of $485.25 ($46.10)
18% of $639.40 ($115.09) 20 1/4% of 840.00 ($170.10)

C. The mathematics is straight forward. Concentrate on practical examples of discounts and their applications. Investigate prices on a sales flier and determine if the sales price and regular price indicate that a consistent discount is being offered.

D. Encourage the students to calculate the net price from a series of discounts by using a series of multiplication rather than calculating discounts and subtracting; ie. the second of the illustrated method.

14.8 Determining the Rate of Trade Discount

14.9 Cash Discount

A. Objectives - The student can calculate the rate of discount required to give a specified net price.

- Is aware of various types of discounts that may be offered.

B. Express as a percentage

$425 of $15 250	(2.8%)	$32.50 of $635	(5.1%)
$4.20 of $12.75	(32.9%)	$ 2 760 of $42 950	(6.4%)

C. There is no new mathematics in these sections. They give practice in the mechanics of percentage and introduce new applications.

D. Exercise 14-9 Qu. 4 involves calculating the number of days from the billing date to the date of payment. Have the students investigate the various penalties charged for late payment of charge accounts.

14.10 Sales Tax

A. Objectives - The student becomes aware of hidden taxes and their affect on the price.
- Can calculate full cost to the customer.

B. Calculate:

115% of $439	($504.85)	107% of $36.40	($38.95)
110% of $1 460	($1 606)	112% of $750	($840)

C. Note that while discounts decrease the price, tax increases it. If the students remember this simple point they will have little difficulty in deciding whether to multiply by 0.85 or 1.15 in the case of a 15% discount or tax.

D. The exercise includes provincial sales tax which will vary from province to province. Use local data to supplement the material.

Chapter 14 Test

1. Find the gross profit or loss on the following transactions.

   a) 500 pens purchased at 39¢ each and sold for 75¢ each.

   b) 25 lanterns purchased at $7.35 and sold for $12.95 each.

   c) 10,000 metal washers purchased at $12 per 1,000 and sold in packages of 10 for 25¢.

2. Western Fashion Store opened with an inventory of $125,465. Over the next three months they purchased goods at a cost of $75,940. Their inventory at the end of the period was $109,720. Net sales, for the period were $140,683 and expenses were $36,294.

   a) Find i) cost of goods sold
      ii) gross profit
      iii) net profit

   b) Draw up a statement of profit and loss showing cost of goods sold, gross profit, operating expenses and net profit as a percentage of net sales.

3. Handy Hardware supply purchases socket wrench sets at $34.60 and sells them for $62.28.

   a) What is the rate of markup?

   b) What is the rate of margin?

4. Grip Tread Tire Co. gives a trade discount of 15% and a fleet discount of 6%.

(a) If a tire lists at $49.95 what is the net price?

(b) If the tire is subject to 12% federal sales tax and 7% provincial sales tax what is the cost to the customer?

(c) What single discount would replace the 15% and 6% discounts?

Chapter 14 Test Comments and Answers

It is suggested that calculators be allowed for this test.

Answers

1(a) $180, (b) $140, (c) $130 (profits)

2(a) i) $91,685, ii) $48,998, iii) $12,704

Net Sales	$140 683	100%
Cost of Goods Sold	91 685	65%
Gross Profit	48 998	35%
Expenses	36 294	26%
Net Profit	12 704	9%

3(a) 80% (b) 44.4%

4(a) $39.91 (b) 47.83 (c) 20.1%

Review and Preview to Chapter 15

Exercise 1 is a review of basic operations and should be used as required. Exercises 2 and 3 review ideas central to the coming chapters and should be covered.

Chapter 15 Statistic

This chapter should be covered in from 5 to 12 periods including the Review Preview and Testing. (5 periods does not allow for covering grouping data).
Students should have a basic knowledge of statistics if only to become knowledgable consumers. The more complicated mathematics occurs in this chapter where we get into grouped data. The main ideas can be covered without using this technique. Students likely to go on in technology should be exposed to this technique.

15.1 Measures of Central Tendency

A. Objective - The student reviews the ideas of mean, median, mode and range.
- Can calculate these statistics for a given set of data.

B. Investigate the terms mean, temperature, median strip, a la mode.

C. These ideas should not be new to the students but when expressed as formulas they tend to look formidable. Have the formula expressed as words ie., "The median is the middle value if there is one, and the mean of the two middle values if there is an even number of terms."

D. Note that question 9 has data on both pages 323 and 324. For question 11 (c) the first group has the higher median and mean scores therefore they are marginally better.

15.2 Grouping Data To Find the Mean

A. Objectives - The student can organize a set of data by grouping into class intervals and calculate the mean using class mid values.

B. Divide the numbers from 17 to 61 into 9 equal groups. (17-21, 22-26, 27-31, 32-36, 37-41, 42-46, 47-51, 57-56, 57-61)

C. The idea of grouping as an aid to calculation has been outdated by the electronic calculator and computer. It does have value still as a step in graphing the data. Stress the graphing aspect of this topic.

D. The data in the exercises quickly becomes dated. Use the questions as models and have the class collect their own data. Newspaper advertisements are a good source.

15.3 Grouping Data To Find the Median

A. Objectives - The student learns the skills of interpolation to find a median value.

B. Solve $\frac{16 - 12}{x - 5} = \frac{24 - 16}{20 - 8}$ (x=11)

C. This topic has application to finding mean differences in trigonometric tables, and to basic ideas of calculus. The ideas are further developed in the next section on percentiles. Stress the ideas of slope and the approximation of the straight line to a short section of the curve.

D. Questions 1-3 give data that is already grouped and should not present too much of a problem. Questions 4,5,6 require grouping of the data. In question 4 $200 is a reasonable class interval, in question 5 treat the data as discreet, in question 6 $10,000 is a reasonable class interval.

15.4 Percentiles

15.5 Measures of Dispersion, Semi Interquartile Range

A. Objectives - The students can calculate the percentile rank of a score.

- Comes to a better understanding of the percentile scores they may receive.
- Learns the meaning of quartiles and can calculate interquartile range.

B. Change the following fractions to percentages to the nearest 0.1%.

$\frac{56}{120}$ (46.7%) $\frac{84}{209}$ (40.2%) $\frac{125}{156}$ (80.1%)

C. The idea of percentile may be covered as in example 1 P. 332 without using grouped data. This limited approach will still help the student interpret test results given in percentiles.

D. In Ex. 15-4 questions 1 to 5 can be done directly without interpolation and the use of grouped data.

15.6 Measures of Dispersion; Standard Deviation

A. Objectives - The student can interpret the significance of standard deviation and shows this by calculating the number of cases that will lie within one, two or three standard deviations of the mean.

- Can calculate standard deviation for a set of data.

B. Calculate

a) 68.3% of 5 284 (3608.9)
b) 95.5% of 15 162 (14497.7)
c) 99.7% of 4 851 (4836.4)

C. Since the standard deviation is now programmed into many calculators, it is more practical for the student to gain an understanding of the significance of the measure than to gain proficiency in doing hand calculations.

D. Questions 1,2,3,6,7 are directed towards an understanding of the significance of standard deviation.

Chapter 15 Test

1. Find the mean, median and range of the following numbers.

   25, 36, 18, 45, 31, 27, 21, 42

2. Find the median age 16a 4 mos, 15a 10 mos, 16a 9 mos 16a 3 mos., 17a 2 mos., 16a 11 mos., 16a 0 mos., 16a 4 mos., 15a 11 mos., 16a 10 mos.

3. (a) In Canada Fitness Tests a student finished 28th out of 150 students. What percentile was the student in?

   (b) In the flexed arm hang a student has a time that places her in the 65th percentile for girls in her age group. Explain what this means if there were 530 students in the group.

4. A metal stamping machine turns out bolts with a mean length of 80 mm and a standard deviation of 0.4 mm. Complete the following statements.

   (a) 2/3 of the bolts will have lengths between ______ and ______.

   (b) Less than 1% will be shorter than ______ or longer than ______.

5. The ages of students in a grade three class are normally distributed. If there are 8 students with the mean age of 102 mos. and the standard deviation of the ages is 3.0 mos. Sketch the frequency graph of their ages.

Chapter 15 Test Comments and Answers

The test does not include questions requiring grouped data. This topic should be evaluated through projects or assignments.

Answers
1. mean 30.6, median 29, range 27
2. median 16a 4 mos.
3. (a) 82% (b) as good or better than 344 students her age
4. (a) 80.4 mm, 79.6 mm
   (b) 78.8 mm, 81.2 mm

Answers to Marginal Puzzles

p. 4 increases 8 times

p. 6 6 numbers

p. 9 coin

chin

chip

chop

shop

p. 13

			Q				
					Q		
							Q
	Q						
						Q	
Q							
		Q					
				Q			

p. 15 7

p. 18 11

p. 21
753
9753
51
5
10562

p. 25 16

p. 30
763
2884
3647

p. 33 road

toad

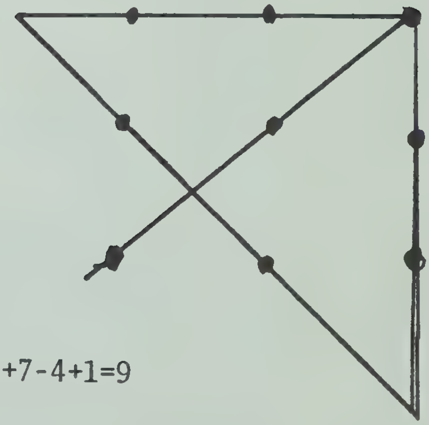
told

toll

tall

p. 36 5.6 decimal point

p. 40



p. 48 3+2+7-4+1=9

p. 49
9567
1085
10652

p. 51 car

tar

tap

tip

p. 54 15 games

p. 57 $233 600.00

p. 59

$$\begin{array}{r} 92836 \\ 12836 \\ \hline 105672 \end{array}$$

p. 69 tool

cool

coil

chin

p. 71

$$\begin{array}{r} 9552 \\ 902 \\ 382 \\ \hline 10836 \end{array}$$

p. 72 $1^3 + 5^3 + 3^3$
$= 1 + 125 + 27$
$= 153$

p. 80

$$\begin{array}{r} 64143 \\ 74115 \\ 9783 \\ \hline 148041 \end{array}$$

p. 81 4,7
3,5
2,3
1,1
0,-1

p. 83 $2 \times 3 + 4 - 5 = 5$

p. 85 $x = 1 \quad y = 1$

p. 92 $12 - 2 \times 6 \times 1 = 0$

p. 110 6 years old

p. 114 area increases 9 fold

p. 121 55, 49, 1045

p. 131 $3 - (2 + 1) = 5 \div (3 + 2) - 1$

p. 134 0.16 cm^2 ie $(\sqrt{3} - \frac{\pi}{2})cm^2$

p. 137

$$\begin{array}{r} 2178 \\ 4 \\ \hline 8712 \end{array}$$

p. 143

$$\begin{array}{r} 341 \\ 27\overline{)9207} \\ 81 \\ \hline 110 \\ 108 \\ \hline 27 \\ 27 \\ \hline 0 \end{array}$$

p. 156 $\frac{9147}{3049} = 3$

p. 168 swim

swam

slam

slat

flat

p. 195 math

mate

gate

game

p. 200 6d T 88

p. 206

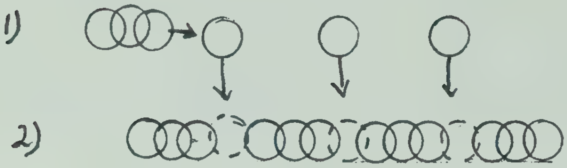
$$\begin{array}{r} 142857 \\ 3 \\ \hline 428571 \end{array}$$

p. 285 $(3^4)^5$

p. 286 $i^n = 1$

p. 289 30 sec.

p. 294 75¢



p. 297 1) stroke out 3 marks
2) leave 4 marks by stroking 1, 2 or 3 marks
3) stroke all remaining marks

p. 300 walk ½ time
bicycle ½ time

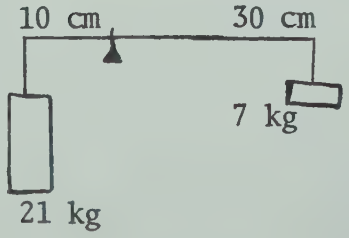
p. 304 111 - 11 = 100

p. 306 48084

p. 311

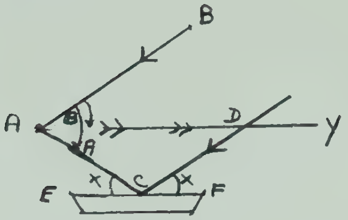
	3	5	8
start	0	0	8
	0	5	3
	3	2	3
	0	2	6
	2	0	6
	2	5	1
	3	4	1
finish	0	4	4

p. 313



p. 315 800 rpm cloclwise

p. 320



since AY ∥ EF then $\measuredangle X = \measuredangle(A - B)$ (1)

and $\angle CDA = X$

since CD ∥ AB then $\measuredangle CDA = \measuredangle B$

$\therefore \measuredangle B = \measuredangle X$ (2)

$\measuredangle X = \measuredangle(A - X)$ [From (1)]

$\therefore \measuredangle A = 2X$

and $\measuredangle A = 2B$ [from (2)]

p. 332 3/8

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